High-Precision Trajectory Determination Using Satellite Navigation Systems

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Abstract

This paper presents the results of ground and fight tests of a prototype flight reference and aircraft tracking system_ Gull has conducted two Sight evaluations of the system in conjunction with Ohio University's Avionics Engineering Center. The Gull system is designed to construct aircraft approach and landing trajectories in flight using the Global Positioning System Position solutions based on interferometric carrier measurements are robust to changes in satellite geometry and multipath The results of over forty approaches confirm that the system meets stringent accuracy criteria required for flight inspection of precision approach and landing systems. The results of this program *are* being applied to Gull's next generation of flight inspection and navigation systems.

Introduction

A Differential Global Positioning System (DGPS) can determine a vehicle's position relative to a fixed site with sub-decimeter accuracy [1-5]. The ground and airborne components of a DGPS contain GPS receivers and telemetry transceivers. A flight reference system combines local GPS measurements with telemetered ground data (Fig. 1), while ground tracking is performed using telemetered airborne data. The most accurate DGPS solutions are based on interferometric measurements of the 1575 MHZ GPS radio-frequency (RF) carrier phase. While the phase of the carrier can be determined with centimeter precision, initial ambiguities in the solutions must be resolved to achieve sub-decimeter positional accuracy [6-8].

Current ambiguity-resolution techniques rely on measurements from a second GPS RF at 1227 MHZ [1,2,4] or on satellite-like transmissions from pseudo-lites placed along the vehicle's route [3]. Recovery of the 1227 MHZ carrier is complicated by encrypted code modulation and by RF interference. Use of pseudo-lites requires additional reference and mobile equipment, and the coverage volume of pseudo-lites is constrained to avoid interference with GPS.

This paper describes a straight-forward approach to high-accuracy trajectory determination using the 1575 MHZ GPS carrier. In this technique, the trajectory is reconstructed backwards in time from a fix point near the runway threshold, with altitude fixed at the threshold by a calibrated altimeter [5]. The runway fix is used to resolve the ambiguities in interferometric solutions. The results of two flight evaluations have established that this technique meets all requirements for flight inspection of Category II landing systems. The results of this program are being applied to a new line of flight inspection products for domestic and international applications.

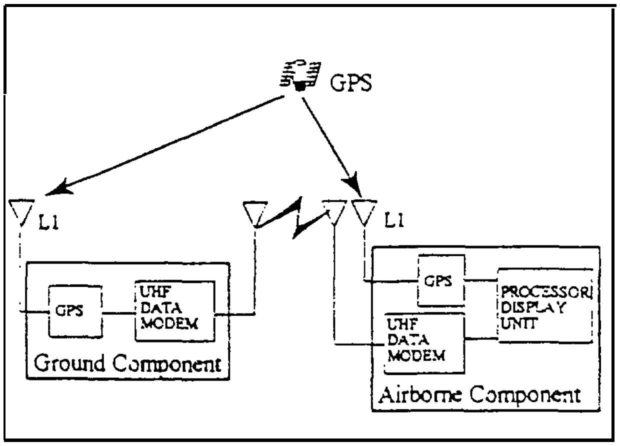

Figure 1. Differential GPS Flight Reference Configuration.

GPS Positioning

C/A Code Positioning

The GPS enables conventional users to determine their position in an earth-centered earth-fixed reference frame, x(t), with an accuracy of about 100 m. Conventional GPS positioning is based on a set of *pseudo-ranges, which* are measurements of the propagation delay of Coarse-Acquisition (C/A) ranging codes transmitted by the satellites [6]. A pseudo-range from satellite "k" to the airborne antenna, p'(t), is related to the true antenna-to-satellite range, $r^k[x(t)]$, by the equation:

$$p^k(t) = r^k[x(t)] + Cdt(t) + e^k(t) \qquad (1)$$

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where C is the speed of light, dt(t) is the receiver clock error, and $e^k(t)$ represents other errors. Because receiver clock error contributes equally to every pseudo-range. it can be removed be removed by differencing two pseudo-ranges. Three position components, s(t) can be solved from three differenced pseudo-ranges from four satellites and the satellite ephemerides. which are transmitted by the safe&s.

The pseudo-range errors, $e^k(t)$, are highly correlated to the errors experienced by the ground receiver. With pseudo-ranges from the ground receiver, $p_g^k(t)$, *double-differenced* pseudo-ranges, P(t), can be defined by:

$$P(t) = p^k(t) - p^j(t) - [p_g^k(t) - p_g^j(t)] \qquad (2)$$

From eq. 1.?(t) can be related to aircraft position, x(t), by:

$$P(t) = r^k[x(t)] - r^j[x(t)] - r_g^k(t) + r_g^j(t) + E(t) \qquad (3)$$

The ground-to-satellite ranges $r_g^k(t)$ and $r_g^j(t)$ are functions of the known pound antenna coordinates. Residual errors, E(t), of one to three meters are caused primarily by multipath from obstructions nearby the antennas. Multipath can be mitigated by receivers with narrow correlator spacing and by siting the ground antenna carefully [9]. Residual atmospheric effects can be compensated for based on models or using supplementary data. When satellite geometry is poor, the effects of these errors on the position solution worsen. Flight test experience indicates the vertical accuracy of code solutions is about three meters with commercial narrow correlator-spacing receivers.

Interferometric Positioning

Significant accuracy improvement is possible using the 1575 MHZ carrier phase. Accumulated carrier phase is the integral of the difference between the recovered carrier from a satellite and the phase of the receiver oscillator. The accumulated phase of satellite carrier "k" at the airborne antenna in cycles, $\phi^k(t)$ is related to r [x(t)] by the relationship:

$$\phi^k(t) = -(1/\lambda)\, r^k[x(t)] + n^k(0) + \epsilon^k(t) \qquad (4)$$

where λ is the carrier wavelength, $n^k(0)$ is an unknown integer constant (i.e, an *integer ambiguity*), and $\epsilon^k(t)$ represents other errors [7]. A double-difference carrier phase, $\Phi(t)$, is formed from local and telemetered carrier-phase:

$$\Phi(t) = -[\phi^k(t) - \phi^j(t)] + [\phi_g^k(t) - \phi_g^j(t)] \qquad (5)$$

Using eq. 4, $\Phi(t)$ can be re-written as:

$$\Phi(t) = (1/\lambda)\{r^k[x(t)] - r^j[x(t)] - r_g^k(t) + r_g^j(t)\} + N(0) + \epsilon(t) \qquad (6)$$

where X(0) is an constant integer ambiguity. Errors $\epsilon(t)$ due to carrier multipath and residual atmospheric effects can be kept to the centimeter !eve!.

Physically, 4(t) can be interpreted as measuring the antennas' relative position within an interference pattern created by the two satellites' carrier waves. Any relative movement of the airborne and ground antennas through the pattern is detected as long as the receivers maintain carrier lock. Position can be determined from three or more such interference patterns created by four or more satellites. The difficulty with these interferometric solutions is that unknown ambiguities in the measurements (i.e., N(0) term in eq. 6) must be resolved to fix the initial position. If the integer ambiguities are resolved (i.e., determined exactly), the position solutions are accurate to within a few centimeters.

Carrier-Smoothing

As a step toward resolving the ambiguities, an ambiguity estimate can be made using filtered pseudo-ranges. Like the code solutions above, a *carrier-smoothed* solution is amenable to real-time computation. Equations 3 and 6 can be combined, yielding:

$$N(0) = [\Phi(t) - (1/\lambda)P(t)] + E_N(t) \qquad (7)$$

The first term is the code-carrier residual, and the second tern represents residual errors on the order of one meter. A real-valued approximation to N(0) based on eq. 7 may be several cycles different than the hue integer value. Because multipath effects tend to average out over rime, an improved ambiguity estimate $\hat{N}(t)$ can be made by passing the code-carrier residual through a low-pass filter, such as:

$$\tau \frac{d\hat{N}(t)}{dt} + \hat{N}(t) = [\Phi(t) - (1/\lambda)P(t)] \qquad (8)$$

Flight tests indicate that the filtered estimate converges to within about three cycles or' the true value of N(0). Real-time aircraft position solutions based on $\hat{N}(t)$ are accurate to well within one meter in the horizontal piane. and within two meters vertically.

Static Tests of GPS Solutions

Figure 2 compares code, interferometric, and carrier-smoothed solutions for the separation of two stationary GPS antennas. The actual antenna separation is 1.1 meters. The code solution exhibits a peak error of about four meters, while the maximum error of the carrier-smoothed code solutions is about two meters. The interferometric solution is accurate to two centimeters since the integer ambiguities are known for this test.

The dynamic racking capabilities of interferometric solutions can be demonstrated by a Circle Test. In the Circle test, a GPS antenna is mounted on a one-ft diameter tumtable and spun. Figure 3 is a plot of the calculated North vs. East position of the moving antenna (each denoted by an "X") compared to the actual antenna psition (the circle). The test was performed at Parker's Gull Electronics Division in April, 1994, using NovAtel narrow-correlator-spacing GPS receivers.

Ambiguity Resolution

The satellite carrier interference patterns shift over time, making isolation of the exact ambiguities possible. Real-valued ambiguity estimates establish a search volume. An ambiguity resolution search involves making several guesses and comparing the consistency of the resulting solutions over time. Redundant satellite measurements speed the search process. Relatively simple *static* ambiguity resolution processes require that both antennas remain stationary for up to several minutes. While a static process could be performed prior to the flight, brief losses in lock by the receiver can cause the carrier phase outputs to shift by a number of cycles. These *cycle slips* invalidate the statically-obtained ambiguities, so 'on-he-fly ambiguity resolution techniques, which resolve ambiguities with one antenna in motion, are required [7,8]. Real-time ambiguity resolution is being pursued as means of meeting Category III landing system requirements, while post-flight software is available commercially for surveying applications.

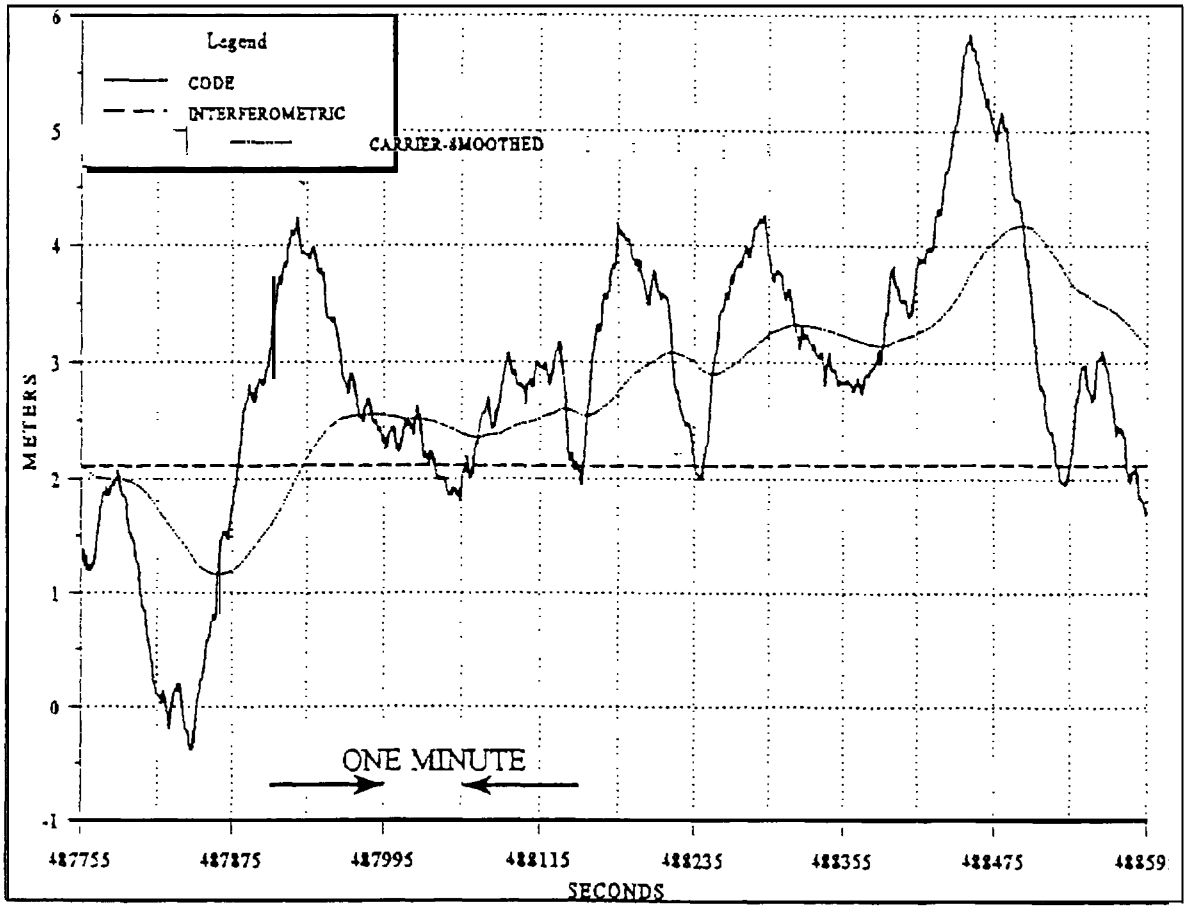

Figure 2. Comparison of Code, Carrier-Smoothed, and Interferometric Solutions.

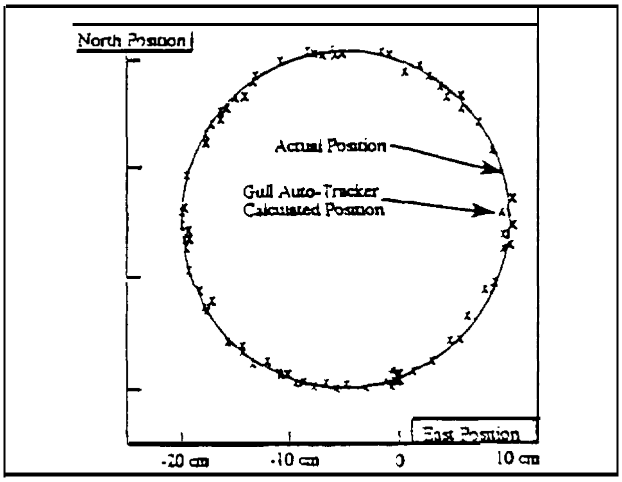

Figure 3. North Vs. East Position for Circle Test.

Airborne Ambiguity Resolution Techniques

Receivers with measurement channels for the second GPS carrier frequency at 1227 MHZ carrier can be combined with the first carrier to form an interference pattern with an 86-cm wavelength The search for ambiguities in the relatively long-wavelength measurements is much faster. The second carrier also can be used to estimate residual ionospheric delays. Once the wide-line ambiguities are determined, a wide-lane interferometric solution is used for a second search which establishes the 1575-MHZ ambiguities. To recover the 1227 MHZ carrier, special ranging codes must be de-modulated. Specially-designed receivers can recover a full-wavelength 1227 MHZ carrier even when it is modulated by encrypted codes. Dual-frequency GPS data from an Ashtech Z-12 system were processed using post-flight ambiguity resolution software to provide truth data for the accuracy analyses of this paper. The Ashtech system and post-flight software have been proven in laser tracker evaluations sponsored by NASA Langley. Ohio University used a real-time version of this approach for demonstrations of aircraft auto-landing.

A different approach which does not require dual-frequency receivers, uses ranging transmissions from the ground instead. Low-power transmitters known as pseudo-lites produce C/A ranging codes at the 1575-MHZ carrier frequency, like the GPS satellites. In addition to a favorable increase in signal redundancy, favorable geometry is obtained by placing the pseudo-lite so that the aircraft flies over it. The approach promotes robust resolution of the ambiguities. Additional required airborne hardware includes a downward-facing GPS antenna. Research programs at Stanford University and elsewhere have established the feasibility of this technique to provide centimeter-accurate navigation

Interferometric Trajectory Reconstruction

Gull has implemented an Interferometric Trajectory Reconstruction (ITR) technique to determine an approach trajectory of an aircraft immediately following a low-altitude pass of the runway. The advantage of reconstructing the trajectory in reverse is that conventional radio altimetry can be used for ambiguity determination, eliminating the need for dual-frequency receivers and pseudo-lite equipment. ITR has been implemented as follows: a real-time navigation solution is propagated using carrier-smoothed GPS solutions as the vehicle approaches a landmark of known height (e.g., the runway threshold). One or more radio altimeter measurements are made over the landmark, which are used to condition the a-priori ambiguity estimates. Corrected solutions also can be computed forward in time. Incorrect integer ambiguity estimates cause a bias and a gradual divergence over time. Alternatively, real-valued ambiguity estimates can be computed directly from the fix data which are of sufficient accuracy for flight inspection.

Flight Test Results

A prototype DGPS flight reference system was assembled and flight tested at the Ohio University Airport in Albany, Ohio, on September 1-3, 1994. The results of the September tests are presented in [5]. A second version of the prototype was brought to Albany and tested on May 9-13, 1995. For the second set of tests, the prototype's airborne module was upgraded with newer GPS receivers and improved positioning software. Data link equipment was provided by GLB Electronics, Inc. The Ohio University's Avionics Engineering Center was contracted to provide an aircraft, pilot, truth system, and technical assistance for the tests. The Center has a twenty-year history of research, development, installation, and flight inspection of navigation aids and avionics systems.

Flight Guidance

Aircraft guidance was provided in several ways during the tests. For some flights, the Center's Piper Saratoga was outfitted with Ohio University's Interferometric Flight Reference/Autoland (IFRA) system [1], which provided vertical and lateral flight guidance. The IFRA used data from Ashtech Z-12 dual-frequency receivers in some cases and single-frequency equipment in other cases. For flights without the IFRA, the university's commissioned localizer and an experimental glideslope were used or the approach was flown visually with occasional call-outs based on Gull's prototype. Dr. Boo Lilley, Director of OUAC, piloted the Saratoga for all flights The flights included five- and tenmile approaches to low-passes and landings. Static data were taken between some runs to facilitate the evaluation of solution drift. Real-time DGPS solutions were tested, and data was logged for post-test analyses. Figure 4 presents the vertical profiles of the second-week tests.

Truth Systems

The OUAC operated two tracking systems to provide truth data: Ashtech Z-12 GPS receivers in the aircraft and on the ground logged data throughout the test period for post-flight DGPS positioning A tracking theodolite also was operated during some of the approaches. The Ashtech DGPS solutions have been used as the primary source of truth data for the accuracy evaluations of this paper. The Ashtech system has been selected because of its proven high accuracy as well as its continuous tracking capability. The Ashtech system has been evaluated at the FAA Technical Center's laser range, confirming its accuracy to the limit of the laser-tracker's performance (about 1 meter) [2]. Ashtech's PNAV post-processing software provides centimeter-level accuracy by resolving the exact integer ambiguities in the L1 carrier data. OUAC, which evaluated the tracker data, found good agreement between the Ashtech DGPS and the theodolite tracking system. The OUAC uses this theodolite system regularly for flight inspection of ILS and Microwave Landing Systems (MLS).

Accuracy Evaluations

Post-processed DGPS solutions of the Parker Gull system have been compared to the Ashtech PNAV solutions. Summary statistics for forty-four approaches from the second test in Table 1. Overall accuracy (2σRMS for all statistics) of carrier-smoothed code solutions is 0.55 meters cross-track and 0.25 meters along-track. Along-track error is somewhat smaller because GPS geometry is more favorable in the East-West direction. Accuracy of ITR solutions over a two-minute propagation time is equal to the fix bias plus a few centimeters. Figure 5 and 6 present the composite residuals of carrier-smoothed code solutions in cross-track and vertical directions. Figures 7 and 8 present composites of the change in residuals of the ITR solutions in cross-track and vertical directions, respectively. The worst-case drift in the vertical carrier-phase solution over sixteen runs is 16 centimeters.

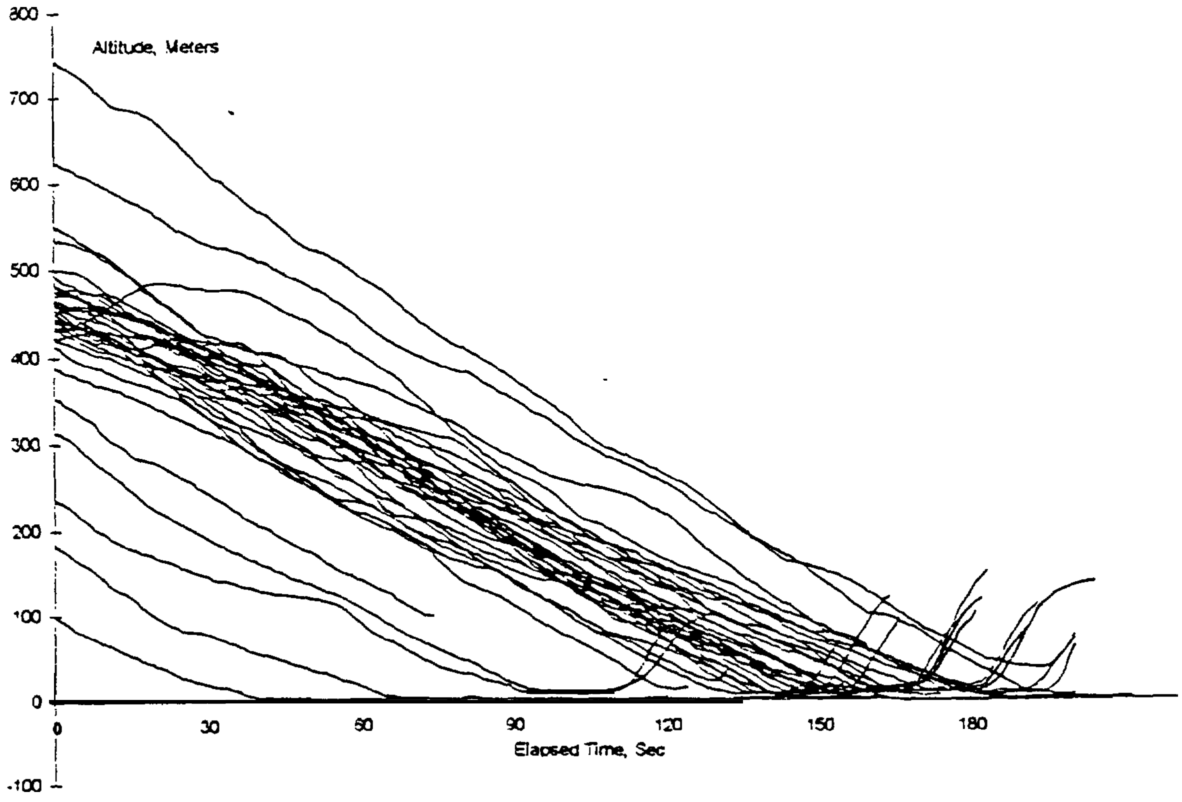

Figure 4. Composite Flight Test Vertical Flight Profiles.

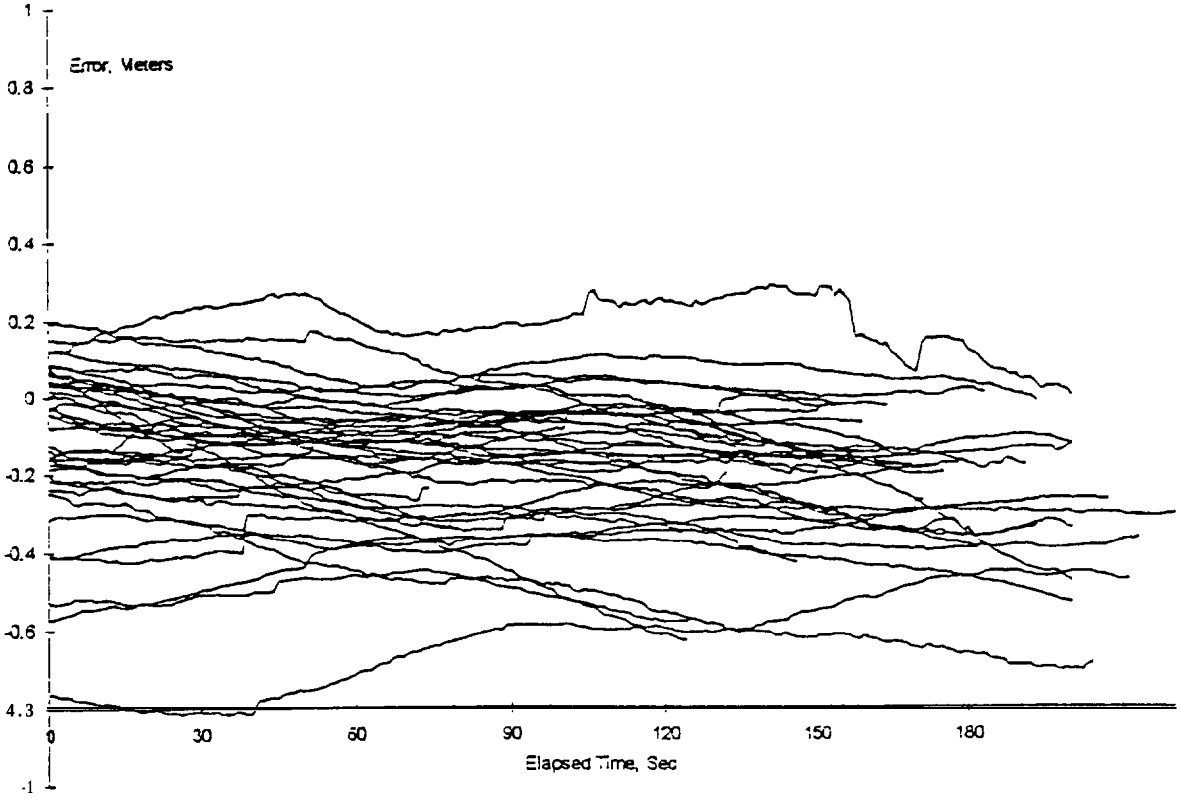

Figure 5. Composite Cross-Track Error of Real-Time Solutions.

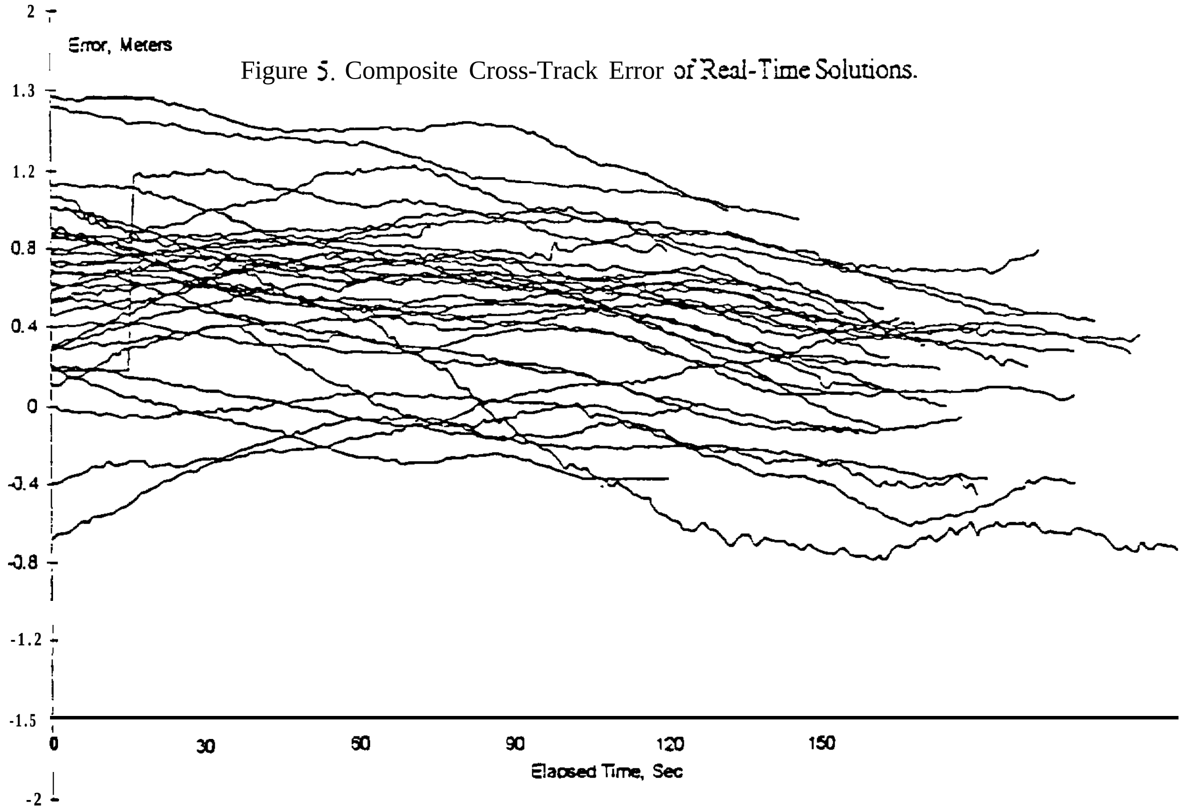

Figure 6. Composite Vertical Error of Real-Time Solutions.

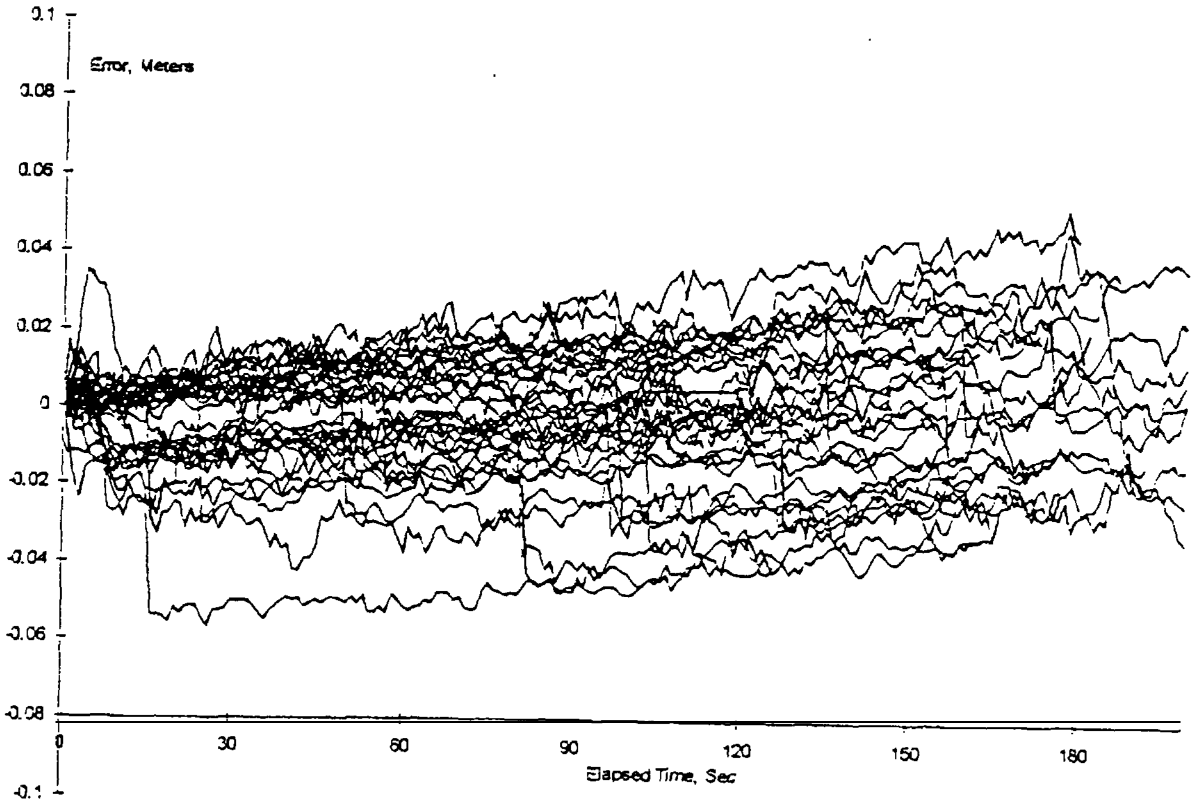

Figure 7. Composite Change in Cross-Track Errors of Interferometric Solutions.

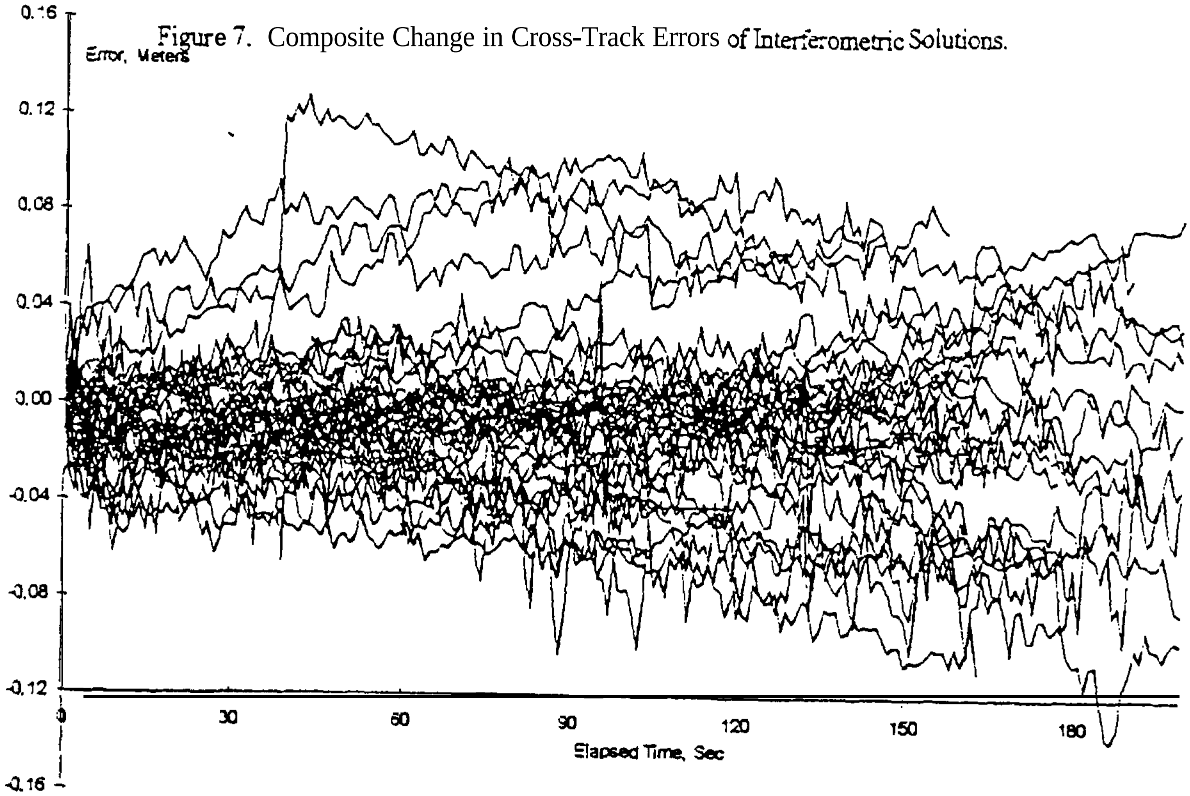

Figure 8. Composite Change in Vertical Errors of Interferometric Solutions.

Solution Evaluated	Cross-track 2σ accuracy	Along-track 2σ accuracy	Vertical 2σ accuracy	Worst-case cross-track error, 44 approaches	Worst-case vertical error, 44 approaches
Real-Time Navigation	55 cm	25 cm	154 cm*	71 cm	194 cm*
Trajectory Reconstruction	55 cm bias ± 6 cm	26 cm bias ±6 cm	Fix bias ± 8 cm	67 cm	Fix bias ± 16 cm

Table 1. Flight Test Results.
("*" indicates quantity not required for ITR)

Interferometric Flight Inspection Systems

The Federal Aviation Administration (FAA) and international agencies perform flight inspection of their radio navigation aids to comply with International Civil Aviation Organization (ICAO) requirements [10]. Flight inspection systems require a tracking capability with an accuracy at least three times better than the landing aid accuracy requirements. Several international governments have adopted optical, laser, and infra-red tracking systems, which have limited range and vulnerability to the weather. With requirements to inspect thousands of radio navigation facilities world-wide, the FAA has abandoned such systems in favor of Automatic Flight Inspection Systems (AFIS). Parker Gull AFIS, which are used for the bulk of all flight inspection in the U.S. use Inertial Reference Systems and other airborne sensors for positioning [11].

ITR technology provides a new alternative for flight inspection that offers many of the advantages of AFIS at lower cost. These results confirm that ITR meets all flight inspection requirements, without inertial systems or survey-grade equipment. Once a base station is established, interferometric techniques can provide the same capabilities as a fully-automatic flight inspection system. Another flight inspection application is for portable tracking systems designed for flight inspection of satellite navigation system approaches [12].

Conclusions

A new approach to precise positioning offers considerable operational advantages for flight inspection and range tracking Combining highly-precise interferometric techniques with established runway fix procedures provides increased robustness and decreased sensitivity to satellite geometry. A system based on ITR can determine a vehicle's trajectory with only secondary use of ranging codes. The size and complexity reductions afforded by ITR may 'be particularly useful in mobile applications requiring both high-accuracy relative navigation and real-time air-ground communications.

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