Improved GPS Positioning for Motor Vehicles Through Map Matching

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Presented at ION-94, Salt Palace Convention Center, Salt Lake City, Utah, Sept. 20 - 23, 1994

BIOGRAPHY

Craig Scott received a BE in Ecectrical Engineering in 1989 from the University of Technology, Sydney. As an undergraduate he worked with the State Rail Authority of N.S.W. and upon graduation, accepted a position as a Senior Tutor with the School of Electrical Engineering at UTS He is currently a full-time postgraduate student at UTS completing his PhD on the incorporation of secondary information sources into positioning systems.

ABSTRACT

Absolute positioning systems, especially the Global Positioning System (GPS), are playing an increasing role in the tracking of motor vehicles. Such systems, however were net designed to specifically track motor vehicles. Apart from the start and finish of journeys, motor vehicles are restricted to the mad network; a fact not reflected by the position measurements. Techniques exist for carrecting the measurements according to a map of the network but a mathematical framework for this correction process is lacking. The author has developed such a framework resulting in a map-aided estimation process that takes into account the measurement noise statistics to optimally translate the raw position measurements onto the road network. The theoratical performance of this estimator is derived and compared with that achieved using GPS measurements. The effect that Selective Availability (SA) has on the map-aided estimator is analysed and a technique for reducing the effect of SA is demonstrated In addition to the accuracy improvements, the map-aided positioning framework also provides a means for readily incorporationg other sources of positioning information. Driver bchaviour, mad type, and vehicle dynamics can be used to make further improvements in positioning accuracy and road identitication.'

1. INTRODUCTION

Motor vehicles are. in general, restricted to travel on roads but GPS and other absolute positioning systems used to estimate **their** position do not inherently have the ability to locate the vehicles onto the roads. The various noise sources that affect the signals and instrumentation used by the positioning system result in the measured position not necessarily lying on the road network Apart from the beginning and end of a journey, a vehicle is highly unlikely to be in the middle of a building as passibly reported by a positioning system and therefore the position estimate needs to be refined to make use of the knowledge regarding the restrictions placed on the vehicle by the road network.

Map matching is well established as a means of utilising map information in positiong systems[3,6]. The effectiveness of the technique being illustrated by the significant proportion of IVHS (Intelligent Vehicle/Highway System) navigation systems that utilize map matching [ll]. Increasingly GPS is being integrated into these systems to further enhance the navigation accuracy. Despite the prevalence of systems using map matching techniques, the method appears to lack a mathematical framework that will ensure the optimal use of the various information sources.

This paper demonstrates a map-aided estimation process within a well defined mathematical framework that allows map informaticn and other sources of position information to be optimally incorporated into a GPS based navigation system. The framework and accompanying estimator are described fully in [15,16j but a summary of this work has been included to assist the understanding of this paper. The work presented represents a part of a larger research program into the general theory of positioning systems [8] and the notation used in this paper is based on this research.

Since the noise in the positioning system results in measurements that do not necessarily lie on the mad, the amount of noise present must be at least the distance from the measurement to the mad. It should be possible to remove this noise to produce a better position estimate. Translating the measurement to the nearest point on the mad network is the intuitive and most straight forward method far achieving this. However, as will be shown, it is not necessarily the optimal position estimate and there is a significant probability that the measurement will be translated onto the wrong road. A MAP (maximum h *posteriori*) estimator has been developed by the author to optimally translate raw position measurements onto the road network by taking into account the properties of the positioning system's measurem ent noise characteristics.

The resultant increase in position es&nation accuracy due to the mapaided position estimator is derived for zero-mean uncorelated gaussian measuremant noise and demonstrated using GPS data collected from static and dynamic (vehicular) platforms. The initial results of the GPS implemantation were clearly affected by Selective Availability resulting in estimation accuracies that, whilst representing all improvement, were not as good as predicted. Anewmodelofthemeasurement noise was implemanted for GPS measurements reducing the effect of SA by treating it as a drift error which is then corrected using standard map matching techniques me SA corrected GPS measurements were processed by the map-aided estimator producing accuracies comparable to that predicted by the theoretical analysis.

2. MAP-AIDED POSTIONNG

In order to utilise road network informaiton for map aided positioning, the maps and the vehicle's domain must be modelled The roads have finite width but the lateral position cn the road is assumed not to be of interest so the road is modelled by its centreline. The restriction of the vehicles domain to the road network 72 will be represented by a uniform positional probability distribution $p(x)$.

$$p(x) = \begin{cases} 0 & x \notin R \\ k & x \in R \end{cases} \tag{1}$$

where k is a constant defined such that $\int_{\mathcal{R}} k dz = I$. The uniform positional PDF implies that no knowledge of the vehicle's behaviour has bun incorporated into the estimation process except for the restricted domain. The addition of supplementary information sources will be discussed later in this paper.

The positioning system makes a measurement y, in the presence of noise η, of the position 2 of a vehicle. In general, positioning systems are affected by a number of independent noise sour-cc so it is reasonable to assume that the noise η has a gaussian distribution which will have a zero mean provided that the system is properly calibrated

$$y = x + \eta \tag{2}$$

$$p(y|x) = \frac{1}{|2\pi N|^{\frac{1}{2}}} e^{-\frac{1}{2}\left[(y - x)^T N^{-1}(y - x)\right]} \tag{3}$$

where

$$N = E\left[(\eta - E[\eta])^T(\eta - E[\eta])\right] \tag{4}$$

In the case of motor vehicle positioning, only two dimensions are of interest The covariance matrix is therefore defined

$$N = \begin{pmatrix} \sigma^2_{\eta_1} & \sigma_{\eta_{12}} \\ \sigma_{\eta_{12}} & \sigma^2_{\eta_2} \end{pmatrix} \tag{5}$$

which can also be expressed in terms of a mean-squared error σ^2 and a matrix describing the geometrical effect of the relative position of the positioning system transponders [12].

$$N = \sigma^2 \begin{pmatrix} \Gamma^2_{\eta_1} & \Gamma_{\eta_{12}} \\ \Gamma_{\eta_{12}} & \Gamma^2_{\eta_2} \end{pmatrix} \tag{6}$$

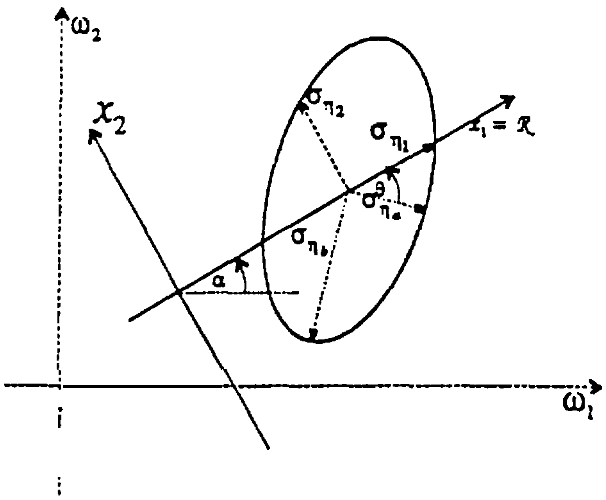

Figure I: Coordinate *frame for map matching: The ellipse represents a* contour *of constant error probability.* σ_{η_1} *and* σ_{η_2}, *are the standard deviations of the measurement error as* **observed in** the X frame. *Similarly,* σ_{η_a} *and* σ_{η_b} *arc the errors observed in the Aframe.*

using [13]. Eq. 3 expands to

$$p(y|x) = \frac{1}{2\pi\sigma_{\eta_1}\sigma_{\eta_2}\sqrt{1-r^2}} \cdot e^{-\frac{1}{2(1-r^2)}\left[\frac{(y_1-x_1)^2}{\sigma^2_{\eta_1}} - \frac{2r(y_1-x_1)(y_2-x_2)}{\sigma_{\eta_1}\sigma_{\eta_2}} + \frac{(y_2-x_2)^2}{\sigma^2_{\eta_2}}\right]} \tag{7}$$

where

$$r = \frac{\sigma_{\eta_{12}}}{\sigma_{\eta_1}\sigma_{\eta_2}} \tag{8}$$

is the spatial correlation coefficient and $|r| \leq 1$.

Having quantitied the assumptions regarding the vehicle's domain and the measurement noise of the positioning system, the estimation process to optimally combine the position measurement with the vehicle's domain can now be defined The MAP estimate of $\hat{x}$ for a position 2 from a given position measurement y is defined by [1,8,101

$$\hat{x} = \arg\max_{x}\left[p(x)p(y|x)\right] \tag{9}$$

3. LONG STRAIGHT ROADS

The simplest scenario for matching a measured position to a map is the case where a vehicle is known to be travelling on a very long straight road. The road $\mathcal{R}$ will be modelled by its centreline and a new coordinate frame $\mathcal{X}$ is defined by translating and rotating the global frame $\mathcal{W}$ such that the road is collinear with the x_1 axis (fig. 1). Thus the road is defined as

$$\mathcal{R} = \{(x_1, x_2) : x_2 = 0\} \tag{10}$$

The positional PDF *p(z) is given* by equation (1) where for this example, k is a small positive constant The position measuremcnt and error covariances measured in the global frame $\mathcal{W}$ are transformed into the $\mathcal{X}$ frame. The MAP position estimate can now be determined by

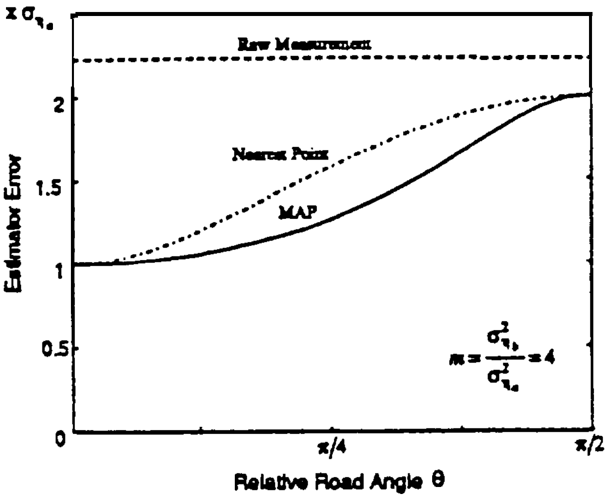

Figure 2: Pcrfomancc of MAP & NP estimators and the raw measurement as a function of road heading (or more specifically, the angle between the A and $\mathcal{X}$ frames. The symmetry of tk equations meant that estimator errors only needed to be plottedfor $0 \leq \theta \leq \frac{\pi}{2}$

solving equation (9) [15].

$$\hat{x} = \left(y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2, 0\right) \quad (11)$$

$$= \left(y_1 - \frac{\Gamma_{\eta_{12}}}{\Gamma_{\eta_2}} y_2, 0\right) \quad (12)$$

The estimated position is easily implemented as it is only a function of the measurement and the relative geometry of the positioning system. It is independent of the magnitude of the errors.

Analysis of this estimator proves it to be unbiased and the variance is given by

$$\sigma_{\hat{x}_1}^2 = (1 - r^2)\sigma_{\eta_1}^2 \quad (13)$$

As mentioned previously, the minimum amount of measurement noise present can be determined by the distance between the measured position and the nearest road. Thus the nearest point (NP) estimator is defined by transbring a given position measurement to the nearest point on the road network. By again using the local coordinate frame X,theNP estimator is described by

$$\bar{x} = \arg\min_{x} \left[(y - x)^T (y - x)\right] \quad (14)$$

$$= (y_1, 0) \quad (15)$$

This is also an unbiased estimator but the variance of **the estimate is larger than the optimal** estimate from the MAP estimator

$$\sigma_{\bar{x}_1}^2 = \sigma_{\eta_1}^2 \quad (16)$$

The difference between the optimal MAP estimate and the more intuitive NP **estimator arises from the MAP** estimator's utilisation of the spatial correlation of the measurement errors As the correlation increases, the **MAP estimator variance decreases eventually becoming** the ideal **estimator when the errors are fully correlated** $(r = \pm 1)$. To evaluate the estimator's true performance, the source and effect of the error correlation need to be investigated.

In two dimensions, normally disuibutcd measurement errors result in elliptical contour of constant error probability. For a given position, the orientation and eccentricity of this ellipse are determined by analyzing the positioning **system in the world coordinate frame** $\mathcal{W}$. The spatial correlation arising fiom this elliptical contour depends **on the heading of the road with respect to the ellipse. In the special case where the ellipse is a circle, there is no spatial correlation irrespective of road heading.** For a general cilipse, the spatial ellipse is zero only when **the road is parallel to either axis of the ellipse.** 'The effect **of the correlation as a function of road heading must be** determined.

Given a measurement cmr distribution (in $\mathcal{W}$) for a given position. it is easy to determine the angle the error ellipse axis makes with the world coordinate frame. From this the length of the minor and major axes can be determined, denoted σ_{η_a} and σ_{η_b}. The angle between the minor axis and the mad heading is denoted as θ (figure 1) and referred to as the relative road heading. The difference between the relative road heading and the uue road heading α is determined by the orientation of the error ellipse. As the relaitve road heading changes the error distribution seen by the road coordinate frame $\mathcal{X}$ changes. By using standard coordinate transformation techniques it can be shown that these **error** parameters as a function of 3 are [15]

$$\sigma_{\eta_1}{}^2 = \sigma_{\eta_a}^2 \cos^2\theta + \sigma_{\eta_b}^2 \sin^2\theta \quad (17)$$

$$\sigma_{\eta_2}{}^2 = \sigma_{\eta_a}^2 \sin^2\theta + \sigma_{\eta_b}^2 \cos^2\theta \quad (18)$$

$$\sigma_{\eta_{12}} = \sin\theta\cos\theta(\sigma_{\eta_b}^2 - \sigma_{\eta_a}^2) \quad (19)$$

$$r = \left(1 + \frac{4m}{(m-1)^2}\mathrm{cosec}^2 2\theta\right)^{-\frac{1}{2}} \quad (20)$$

$$m = \sigma_{\eta_b}^2 / \sigma_{\eta_a}^2 \quad (21)$$

The performance of a positioning system is often quantoficd by the circular amr probable (CEP) [19] but in this instance, the measurements are not distributed in two di**mensions so the RMS error between the estimate and the** true position $\hat{x}$ is more suitable. The MAP and NP es**timators are unbiased and therefore, the RMS errors are** equal to the estimator standard deviations.

$$c(2) = \sqrt{E\left[(\bar{x} - \hat{x})^T(\bar{x} - \hat{x})\right]} \quad (22)$$

$$= \sigma_{\eta_1}\sqrt{1 - r^2} \quad (23)$$

$$e(5) = \sigma_{\eta_1} \quad (24)$$

$$e(y) = \sqrt{\sigma_{\eta_1}^2 + \sigma_{\eta_2}^2} \quad (25)$$

Using Eqs. (17)-(21), these RMS errors are now plotted as a function of the road heading (figure 2). The MAP estimator is clearly a better &rnatnrthantheNPcstimator, equivalence only occurring when the **errors are uncorrelated** ($\theta = 0, \frac{\pi}{2}$).

The RMS estimator amrs quantify the performance of

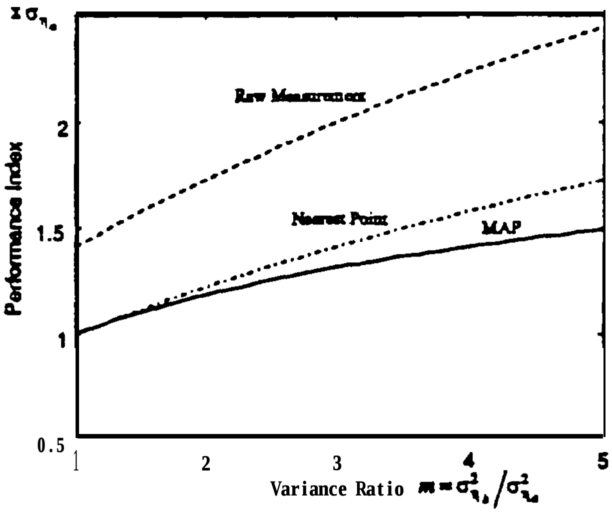

Figure 3: *Estimator performance indices (Eq. 28-30) are plotted as* a function of the measurement noise *ratio m.*

the estimators as a function of the road heading for a given positioning system geometry and measurement noise but they do not provide a means of evaluating the performance of the estimators across an entire network. Since the road headings across an entire network can be assumed to be distributed uniformly an average performance can be determined for each estimator given the raw measurement errors The performance index i is defined below as the RMS error with respect to road heading.

$$i^2(x) = E\left[e^2(x)\right] \qquad (26)$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} e^2(x)d\theta \qquad (27)$$

$$i^2(\hat{x}) = \sigma^2_{\eta_a}.\sqrt{m} \qquad (28)$$

$$i^2(\tilde{x}) = \sigma^2_{\eta_a}\frac{m+1}{2} \qquad (29)$$

$$i^2(y) = \sigma^2_{\eta_a}(m+1) \qquad (30)$$

The index for each esimator is plotted as a function of the measurement error variance ratio m in figure 3. The MAP estimator is clearly the better estimator, it's relative performance increasing as the spatial correlation of the data increases.

4. FINITE STRAIGHT ROADS

The above analysis was for very long straight roads but in an urban area, the roads cannot necessarily be considered long with respect to the positioning system errors. A map-aided position estimator for a road of finite length is required. A straight road $\mathcal{R}$ of length $2l$ is defined by

$$\mathcal{R} = \{(x_1, x_2) : -l \leq x_1 \leq l, x_2 = 0\} \qquad (31)$$

The vehicle's existence on this road is represented by a uniform PDF (Eq. 1).

$$p(x) = \frac{1}{2l}\left[u(x_1 + l) - u(x_1 - l)\right]\delta(x_2) \qquad (32)$$

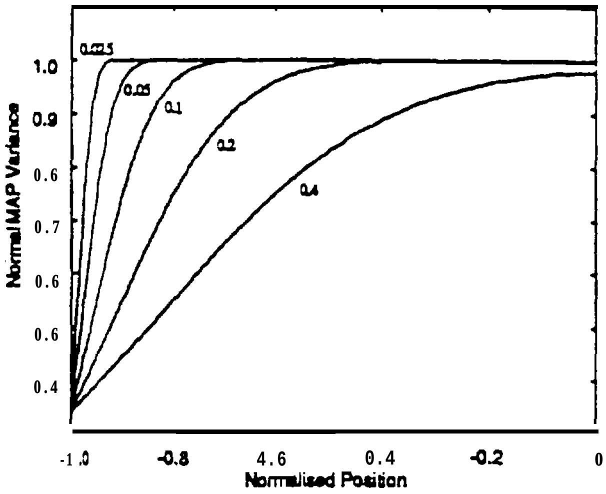

Figure4: *MAP estimator variance on a finite road normalised to the that of a very* long road. *Each trace represents a* different measurement error standard *deviation to length ratio.*

where $u(t)$ is the unit srep function. Using the same measurement noise distribution as for the very long road, the MAP and NP estimates can be shown to be [15]

$$\hat{x} = \begin{cases} (-l, 0) & \text{for } y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2 \geq l \\ (y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2, 0) & \text{for } \left| y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2 \right| < l \\ (l, 0) & \text{for } y_1 - \frac{\sigma_{\eta_1}}{\sigma_{\eta_2}} r y_2 \leq l \end{cases} \qquad (33)$$

$$\tilde{x} = \begin{cases} (-l, 0) & \text{for } y_1 \leq -l \\ (y_1, 0) & \text{for } |y_1| < l \\ (l, 0) & \text{for } y_1 \leq l \end{cases} \qquad (34)$$

The normalised variance of the MAP estimators is plotted in figure 4 as a function of position. Each trace represents a different measurem ent-error to road-length ratio which is effectively a measure of the relative length of the road; the greater the ratio the shorter the relative length of the road Position measurements that lie a significant distance away from the endpoints result in MAP and NP estimates that are no different to those presented earlier for very long roads. When the vehicle is closer to the endpoints, there is an increased possibility that the measurements will lie beyond the road endpoints resulting in some measurements being translated to the nearest endpoint The resulting error is smaller than would have occurred had the vehicle been on a long road. The finite length of the road is a source of position information for both estimators with the MAP estimator again being the better of the two through the utilisation of the measurement error correlation

5. CURVED ROADS

In an urban area the majority of roads are straight but the curved roads must still be included in a map-aided position estimation system. The process to determine the MAP estimate for the curved road $\mathcal{R}$ is similar to that for a straight road. The curved road is defined by:

$$\mathcal{R} = \{(x_1, x_2) : x_2 = \rho(x_1)\} \qquad (35)$$

where $\rho(x_1)$ is a function that models the road's centre-line to a desired accuracy.

The knowledge that the vehicle lies on the road is again represented by a uniform probability disuibution (Eq. 1) where k is the inverse of the length of the road.

$$k = \left[\int_{x_{i_1}}^{x_{f_1}} \sqrt{1 + [\rho'(x_1)]^2} dx_1\right]^{-1} \quad (36)$$

A number of possible functions suitable for modelling roads wae investigated (see [15],[16]) but the current digital map standards (for example GDF [4]) and map accuracies implied that a piecewise linear road model would be no less accurate than the higher order models. This model also means that the entire road network can be described by a single component the finite straight road.

As illustrated in figure 5, the curve Ξ is modelled by a sequence of linear splines functions $\rho_i(x_1)$ between the positions ξ_{i1} and $\xi_{(i+1)1}$.

$$\rho(x_1) = \begin{cases} \rho_1(x_1) & \text{for } \xi_{11} \le x_1 < \xi_{21} \\ \rho_2(x_1) & \text{for } \xi_{21} \le x_1 < \xi_{31} \\ \vdots & \\ \rho_n(x_1) & \text{for } \xi_{n1} \le x_1 < \xi_{(n+1)1} \end{cases} \quad (37)$$

The linear splines are described by

$$\rho_i(x_1) = m_i x_1 + b_i \text{ for } \xi_{i1} \le x_1 < \xi_{(i+1)1} \quad (38)$$

where

$$m_i = \frac{\xi_{(i+1)2} - \xi_{i2}}{\xi_{(i+1)1} - \xi_{i1}} = \tan\theta_i \quad (39)$$

$$b_i = \xi_{i2} - m_i \xi_{i1} \quad (40)$$

To find the MAP position estimate (Eq. 12), each spline of the curve is treated as a separate road and a locally optimum position estimate $\hat{x}_i$ is determined for that spline. The overall position estimate is then determined by evaluating each local estimate to determine the optimal estimate for the entire curve.

$$\hat{x}_i = \arg\max_{x} p_i(x)p(y|x), i = 1,\ldots,n \quad (41)$$

where

$$p_i(x) = \begin{cases} k_i & \xi_{i1} \le x_1 < \xi_{(i+1)1}, x_2 = \rho_i(x_1) \\ 0 & \text{elsewhere} \end{cases} \quad (42)$$

$$k_i = \left[\int_{\xi_{i1}}^{\xi_{(i+1)1}} \sqrt{1 + [\rho_i'(x_1)]^2}\right]^{-1} \quad (43)$$

$$\hat{x} = \arg\max_{x} p(x)p(y|x) \quad (44)$$

$$\hat{x} = \arg\max_{\hat{x}_i} p(\hat{x}_i)p(y|\hat{x}_i) \quad (45)$$

The MAP and NP position estimates for a given spline are

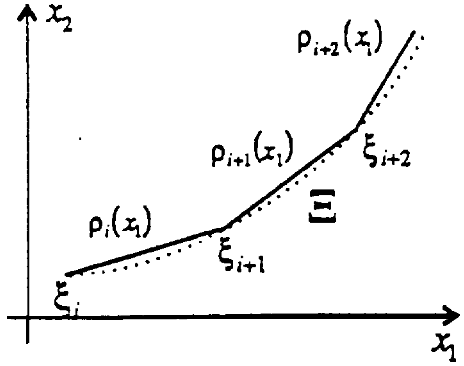

Figure 5: Piecewise road model $\rho(x_1)$ is shown in contrast to the true road path Ξ.

given by finite road estimators (33) and (34) respectively. The performance of the MAP and NP estimators on a piecewise linear road will be comparable to that of the finite straight road except that the endpoint effects will only be noticeable at the curve endpoints and not at every node of the piecewise Curve. The only new problem introduced by curved roads is the possibility of ambiguous and near ambiguous position estimates which arises from the likelihood function $p(y|x)$ having multiple maxima. The reelution of ambiguous estimates, if and when they occur is addressed in detail in Scott [15].

6. ROAD NETWORKS

A road NEIWORK is a set of roads along with a set of nodes which join the various roads together. As this is very similar to the piecewise linear curve model, the same MAP technique CD be used (Eqs. 41-45) where $\rho_i(x_1)$ represents the ith road of the network. The resultant MAP position estimate is optimal but the two-dimensional nature of the network introduces, for the same reasons as for curved roads, the problem of ambiguous estimates. The optimal estimate may actually be on the wrong road resulting in a position estimate further from the true position than the raw measurement. The local estimate on the correct road is a better estimate but it was slightly less likely and hence not chosen. The problem lies in determining which road the vehicle is travelling on. The solution lies in the use of additional sources of information.

The trajectory formed by the most recent raw position measurements can be used to determine which road the vehicle is travelling on and to provide a *priori information* on the next measured position of the vehicle. For absolute positioning systems producing a sequence of position measurements the optimal method for incorporating the measurements and the vehicle's dynamics is the Kalman filter [2]. Normally, two decoupled filters would be required in order to model a motor vehicle's two degrees of freedom but the map-aided positioning estimator results in all measurements lying on known curves and thus the vehicle is reduced to a single degree of freedom. Scott [15,16] describes the required coordinate transformations.

The above technique introduces a circular reference which restricts its effectiveness. The Kalman filter while

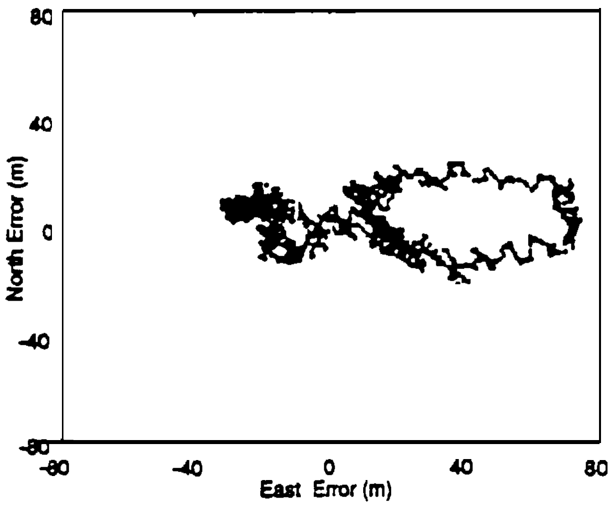

Figure *6: GPS positioning errors* using *the SPS (Data Set No. 2)*

producing a more accurate position estimate isalsobe ing used to identify the road upon which the vehicle is travelling The problem lies with the need to translate the measured position onto a road before the spatially **reduced Kalman filter can be used. To resolve this para-**dox, the author proposes to treat each road of the network as though the vehicle were currently travelling on it The MAP map-aided estimator in determining the best posi tion estimate already maps each position measurement onto every road (restricted to nearby roads in practice) of the network Using track splitting techniques designned for tracking objects obscured ny measurement clutter [l], a Kalman filter is instantiated for each road using the road's own sequence of measurements as the filter input The variance of the kalman filtered position estimate then gives a measure of the likelihood that the vehicle is travelling on that road. Combining this with the likelihood function of the measurement translation provides a means for identifying which road the vehicle is currently travelling on. In this manner, there is a much reduced probability of ambiguous estimates and the plotted position of the vehicle will not be seen "jumping" from toad to road.

7. STATIC GPS ANALYSIS

The Standard Positioning Service (SPS) of GPS was chosen to demonstrate the practical implemention and to evaluate the real performance of the mapaided estimator The increasing use of GPS in vehicle tracking applications [l l] makes it particularly relevant while access to all aspects of the position computation and access to Differential GPS (DGPS) make performance calculations straight forward. THe only problem with applying the map-aided position estimator to GPS is that the errors are temporally correlated which violates an implicit assumption of the MAP estimator **This correlation is pre-**dominantly due to the effects of Selective Availability [9].

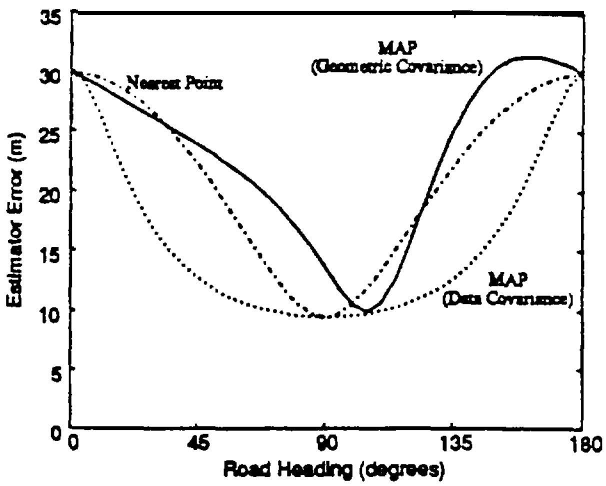

Figure 7: *Performance* of *the map-aided estimator on an arbitrary road* **through** *the* **reference** site *using SPS data (Data Set No. 2). The road* **heading** is *in degrees* **north of East.**

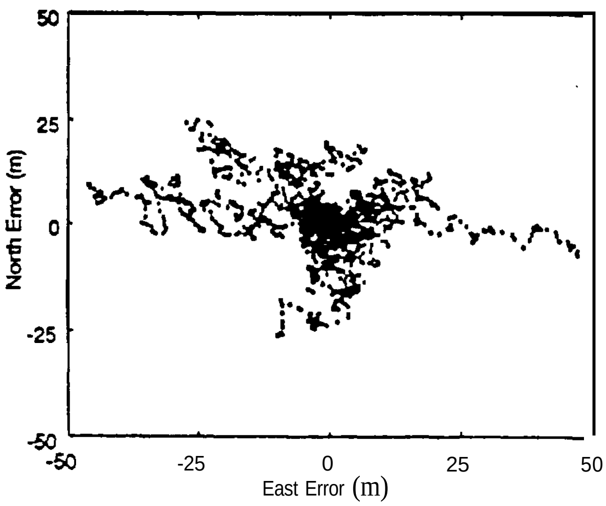

Figure 8: *GPS positioning errors using the SA corrected (DGPS)* data with a 60s update *rate* ***(Data*** *Set No. 2)*

7.1 Experimental Setup

The practical implementation and evaluation of the map **aided estimator does not require dynamic GPS data – an arbitrary straight road can be deemed to pass through a** known point where static GPS dam has been collected. As previously discussed curved roads are modelled by piecewise linear splines and therefore a single straight road test is sufficient The arbitrary road wa treated as though it were infinite as urban roads are relatively long with respect to GPS errors.

A Trimble Accutime 6-channel GPS receiver was situated on a surveyed site with an unobstructed view of the sky and position data was collected Position samples were generated approximately twice a second using only the best 4 satellites to ensure that the effective positioning system geometry was as constant as possible. A Novatel **10 channel receiver at the same site was used to collect the** satellite **ephemeris** data which facilitated the calculation of the geometrical component of the error covariance matrix (Eq. 6) for each of the position samples.

	SPS		SA Corrected (60s Update)		SA Corrected (30s Update)	
	Set	Set 2	Ser 1	Set 2	Set 1	Set 2
Mean HDOP	1.6	1.5	1.6	1.5	1.6	1.5
Mean (HDOP) Error Angle	162”	34.6’	16.2°	34.6”	16.2°	34.6°
Statistical (Data) Error Angle	21.7”	883”	40.5°	752”	38.3”	70.4”
GPS RMS Error (m)	35.0	313	19.0	16.0	105	9.3
NP RMS Error (m)	24.7	21.0	13.4	11.1	7.4	6.4
MAP (HDOP) RMS Error (m)	253	22.8	132	11.0	73	6.3
MAP (Data) RMS Error (m)	24.5	155	13.0	102	72	5.9

Table 1: *Static GPS analysis of map-aided positioning using the result of two GPS data collection experiments. The results were analysed using two sources of covariance information – tk HDOP/satellite geometry and statistical analysis of the GPS errors.*

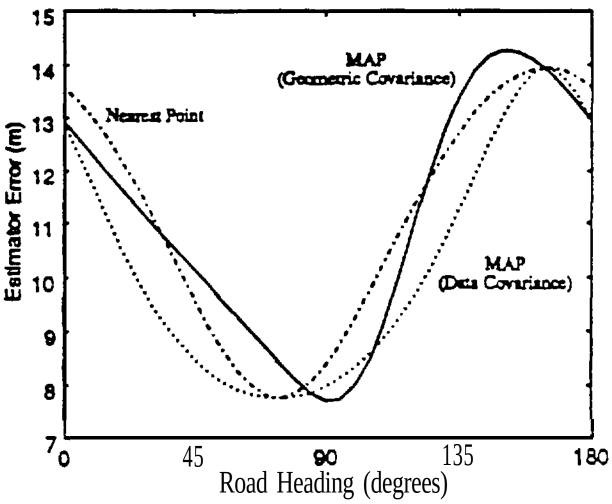

Figure 9: ***Performance of tk map-aided** estimator on an arbitrary **road through tk reference site using SA** corrected data with a 60s update rate (Data **Set No.** 2). **Tk road** heading is **in degrees north of East.***

Two sets of data were collected, each representing a different set of 4 satellites and each comprising a half hour of observations, approximately 1500 samples. As summarised in table 1 the observed positioning errors are consistent with the expected errors given the Horizontal Dilution of Precision (HDOP) [5]. There was however, a significant difference between the orientation of the error ellipse as expected from the satellite geometry and that determined by a direct covariance calculation using the measured errors. ‘Ibis difference is due to the biasing effect of Selective Availability which is clearly evident in a plot of the second data set (figure 6).

72 Map-tided Estimator Performance

The two data sets were processed by the map-aided position estimator to produce three distinct estimates. *The* MAP_{HDOP} estimate implemented the MAP estimator (Eq. 12) using the measurement error distribution as determined from the satellite geometry. The MAP_{Data} estimate is the same except that it uses the error distribution generated directly from the measurement data Finally, the NP estimate is the nearest point estimate (Eq. 15) which is independent of the error distribution.

The performance of each of these estimators was andysed as a function of the road heading, the results of which are plotted for the second data set in figure 7. For each estimator, the RMS error averaged over the range of road headings is given in Table 1. The estimators performed as expected with the exception that the effect of the SA bias meant the best performance occurred for the road heading that matched the calculated error orientation and not the expected orient&on. For both data sets, the MAP estimator performed best (and close to the theoretical predictions) when given the most accurate information regarding the error distribution; but even when using an incorrect error model, there was still sufficient spatial correlation evident for the MAP_{Data} to perform better than the NP estimator. Better relative performance increases would be expected for worse satellite geometries. The data collected was generated by good geometries and consequently the error ellipse was not as elongated as it might be and consequently the degree of spatial correlation was lower.

73 Reducing Effects of SA

The biases intrcduced by SA are clearly responsible for the distortions seen in the measurement distributions. To optimise the performance of the map-aided estimator, a new measurement error model is required that more accurately reflects the biases introduced by Sk In the long term, SA biases reduce to approximately a gaussian measurement error [7], but in the short term the effect of SA can be viewed as a slowly varying bias. As such, it can be subtracted from position measurements provided that a measure of the bias can be made at regular intervals. This is the fundamental principle of Differential GPS (DGPS).

Real time DGPS for vehicular use requires that the differential correction be transmitted to the moving vehicle [14]. This requires a significant amount of infrastructure as well as another piece of equipment in the vehicle. Instead of implementing such a system, corrections can be generated by using map matching techniques similar to those used to correct Dead Reckoned and inertial navigation systems [3, 6]. By analysing the map-aided position estimates, points where the vehicle turned can be

	SPS	SA Corrected
GPS RMS Error (m)	54.6	31.6
NP RMS Error (m)	41.8	16.3
MAP RMS Error (m)	26.8	13.4

Table 2: *Results of applying the map-aided estimators to GPS &a collected from a vehicle moving along a straight mad for 80 seconds. Tk SA corrected data war generated by a* using *a single differential correction generated at the turning point at tk start of the road.*

identified. Curve fitting techniques can be applied *to the raw* measurements to determine the equivalent measurement for the vehicle's turning point which can then be compared with the known turning point generated by map marching to determine a correction vector (figure 10).

To analyse the potential benefits of this form of differential correction, two analyses were performed using the static GPS data using different update rates; once every 60 seconds, and once every 30 seconds. These update rates are much lower than standard DGPS update rates (10s intervals) but they are more indicative of the update rates achieved using the map matched derived corrections; corrections only being available when the vehicle's heading changes significantly. The analysis performed on the SPS data was repeated for the corrected data with the results plotted in figures 8 and 9 and summarised in table 1. The improvement in the GPS measurements are comparable to the expected results [5] with a subsequent improvement in the performance of the map-aided estimators. The reduced effect of SA resulted in the performance of the MAP_{HDOP} estimator becoming closer to that obtained from the direct covariance calculation.

8. DYNAMIC GPS ANALYSIS

The static GPS experiment results confirmed that the map-aided estimators gave significant improvements over the raw GPS measurements. similar performance for a dynamic platform cannot be assumed. A GPS receiver mounted on a motor vehicle is subjected to a number of influences that were not present in the static experiment, particularly signal blockage and multipath due to trees and urban canyons. In addition, the differential signal correction will have to be determined from map matching techniques rather than by direct calculation using a reference point. An experiment was undertaken to confirm that the proposed techniques will work effectively for GPS position measurements made by a moving motor vehicle.

A Trimble Pathfinder Basic Plus 6-channel receiver was fitted to a motor vehicle which was driven around the eastern suburbs of Sydney whilst logging position data A Novatel 10-channel PC-based receiver was located at a reference site to record the satellite observations for DGPS post- processing and to calculate the geometric component of the error covariance. The differentially corrected positions were calculated to provide an accurate estimate of the vehicle's position for error determination and after further curve fitting, to provide a source of digital map information.

The resulting data was plotted in order to examine the effects of SA. In the majority of cases, the effect of SA manifested as a bias implying that the map matching technique would provide a means of improving the position estimation accuracy. The exception to this behaviour occurred on roads where trees or tall buildings or both resulted in a the visible satellite constellation constantly changing; thus preventing the bias of SA from being determined.

A straight section of road exhibiting a consistent satellite constellation *was* then chosen *to* test *the* dynamic performance of the map-aided estimators. The section of road was modelled by fitting a straight line to the appropriate segment of the DGPS data with the actual location of the vehicle being taken as the nearest point from a given DGPS measurement to this road model. In accordance with the map-aided estimator's formulation, the SPS position data and error covariance information were then transformed into a new coordinate frame such that the road ran along the x_1 -axis. The error covariance gave a HDOP of 1.8 and the distribution made an angle of 64.1° with the x_1-axis. The MAP and NP estimators were applied to the data, the results, when compared with the DGPS positions, give the RMS errors expressed in table 2. The trend expressed by the theory and confirmed by the tic GPS results is also exhibited by the dynamic results, albeit only a relatively small sample.

The map matching technique for differentially correcting SA errors was also implemented on the same section of mad. As illustrated by figure 10, the GPS positions measured either side of the intersection are fitted using least squares to lines parallel to the roads being travelled. The intersection of the Iwo lines of the trajectory define the corner for the measurement sequence. The differential correction was then determined by comparing the intersection on the map with that determined form the GPS measurements. This correction was applied to the raw SPS data prior to translation into the road based coordinate frame. The results of the correction process and subsequent map-aided positioning are given in table 2. Again the map-aided estimator performance reflected the theoretical results providing significant accuracy improvements. Although, without the map-aided estimators, the map matched based differential correction on its own provided a significant accuracy improvement.

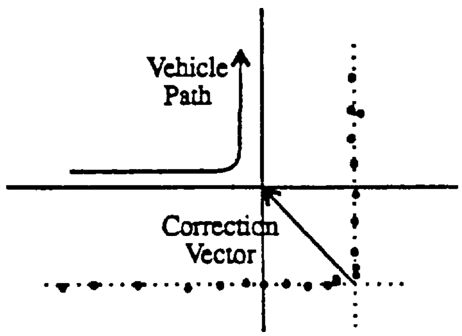

Figure 10: *The* use *of standard map matching techniques to determine a differential GPS correction.*

9. FUTURE IMPROVEMENTS

The author has already identified a number of methods for further improving the map-aided position estimators and for strengthening the underlying positioning system framework. While some of these ideas are for the sole use of GPS based tracking, others are equally applicable to au absolute positioning systems.

9.1 Road Identification

A key component of the map-aided estimator is correct road identification. All of the performance figures derived for the estimator are not applicable if the vehicle has been projected onto the wrong road Therefore, optimal operation of this estimator relies on a robust method for ensuring that the correct road has been identified. As previously mentioned the spatially reduced kalman filter is capable of providing road identification information as well as an improved position accuracy. The author intends to further refine the technique for integrating the kalman filter and the map-aided estimator. In addition, the map-aided estimation framework will be expanded to include map based velocity correction. This will allow velocity measurements to be included in the spatially reduced kalman filter.

A problem with this kalman filter approach arises when the measurement errors are biased either directly by error sources such as SA in GPS or indirectly through map registration errors. It is possible that in these situations, the bias will be sufficient to cause the measured vehicle trajectory to best match with the wrong road. The effectiveness of the differential technique previously described slowly degrades as the SA changes. Lateral errors are. easily removed by the map-aided positioning but any change in the longitudinal bias will remain undetected. If the longitudinal error grows sufficiently large, the wrong intersection could be identified at the next turning point which in turn compounds when the differential correction is determined. To avoid this problem, the SA correction model needs incorporate a drift rate and more sophisticated map matching techniques [3] employed to minimise the risk of incorrect road identification. The kalman filter can also be improved by modifying the system model to incorporate correlated noise and biases [18].

A further technique, specific to GPS and yet to be experimentally verified, is to utilise all visible satellites to calculate a set of position estimates at any given epoch by taking all possible combinations of 4 satellites. Generally a Least Squares approach is used to determine the best position estimate using all visible satellites but the SA effects are not a Gaussian noise and as a consequence the Least Squares estimate may not represent the optimal use of the available information. It may be possible to utilise the spread *of* measurements for a given position to better determine the correct road.

9.2 Supplementary Information Sources

The performance of the MAP map-aided position estimator is easily improved through the incorporation of additional information sources such as:

Road Type: Each road type is assigned a probability according to it's traffic flow capability - a vehicle is more likely to be on a major road than the lane running parallel to the major road.

Road Rules: *Any rule restricting the vehicle's freedom*, (e.g. No Right Turn, one way streets, etc) can be modelled by setting appropriate probabilities to zero. Vehicle speeds can also be used to differentiate local roads (low speed limits) from expressways (high speed limits).

Route Knowledge: Direct route knowledge or route preferences as determined from past journeys [17] (driver behaviour) can be used to bias the positional PDFs in a manner similar to the road type above.

The incorporation of this type of information is straight forward as the information is often available with the current digital map formats (e.g. GDF [4]) and it applies uniformly to each road. That is, the positional PDF for a given road is still uniform and as a result the local position estimate (Eq. 41) is unaffected by the additional information. The information, incorporated by modifying the positional PDF of each road, is only used when determining the globally optimal estimate (Eq. 45).

Other sources and forms of information are available, the most notable being the use of traffic flow rates. For example, a vehicle on a given road is more likely to be found at the intersections. This knowledge, and other knowledge of a similar form, can be represented by a non-uniform positional PDF implying that the MAP estimate will need to be derived from first principles. This is not quite the case as it can be shown that a MAP estimate based on a uniform PDF does not lose any relevant position information [15] and consequently the estimate can he refined if and when other information becomes available such as that represented by a non-uniform PDF.

10. CONCLUSION

The proposed map-aided position estimation system based on maximum *à posteriori* principles greatly increases the accuracy of any positioning system used to track objects with a restricted operating domain; most particularly motor vehicles, but also trains and trams. The position estimation process results in all measurements being translated onto a map such that all of the position estimates now lie on known curves allowing a one-dimensional kalman filter to further improve the accuracy incorporating the dynamics of the vehicle being tracked. The mathematical framework of the MAP estimator used also readily allows further sources of information regarding the vehicle's position and possible movements to be optimally incorporated.

The practical implementation of the map matching framework and associated estimator was confirmed via experiments conducted using GPS Standard Position Service data in static and dynamic environments. These experiments also confirmed the estimator's theoretical performance. The biases introduced by Selective Availability were significantly reduced by combining the map-aided estimator with map matching techniques to provide a differential correction.

The overall position estimate, including the map matched differential correction, are determined from straight forward equations. Given access to a suitable digital map of the road network, the map-matched estimator could be easily incorporated within a GPS receiver.

ACKNOWLEDGEMENTS

My appreciation is extended to Mr. Bernie Hirsch and Dr. Chris Rims from the School of Surveying at the University of N.S.W. for the use of their GPS equipment and their assistance in collecting the GPS data I would also like to express my gratitude for the support, encouragement, and patience of my PhD supervisor, *Dr. Chris Drane,* throughout my research.

REFERENCES

[1] Y. Bar-Shalom and T. E. Fortman, *Tracking and Data Association*. Academic Press Inc., 1988.

[2] R. G. Brown and P. Y. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley & Sons, Inc., 2nd ed., 1992.

[3] J. L. Buxton, S. K. Honey, W. E. Suchowerskyi, and A Tempelhof, "The Travelpilot: A second-generation automotive navigation system," *IEEE Transaction on Vehicular Technology*, vol. 40, no. 1, pp. 41–44, February, 1991.

[4] H Claussen and W. Lichtner, "GDF - A proposed standard for digital road maps to be used in car navigation," in Vehicle *Navigation Information Systems Conference (VNIS '89). pp.* 324–330, 1989.

[5] J. R. Clynch, L. R. Gary Thurmond, and R. Schramm, "Error characteristics of GPS differential positions and velocities," in *Proceedings of ION GPS-92, pp.* 969–978, 1992.

[6] W. C. Collier, "In-vehicle mute guidance systems using map matched dead reckoning," in Position *Location* and *Navigation* Symposium *(PUNS '90), pp.* 359–363, 1990.

[7] R Conley, "GPS performance: What is normal?," in *Proceedings of ION GPS-92*, pp. *W-893.1992..*

[8] *C.* R Drane, *Positioning Systems – A Unified Approach.*, vol. 18 1 of *Springer Series* in *Control* and *Information Sciences*. Springer-Verlag, 1992.

[9] Y. Geogiadou and K. D. Doucet, "The issue of Selective Availability," *GPS World.* p. 53, September/October, 1990.

[10] N. Johnson. W. Tang, and G. Howell, "Terrain aided navigation using maximum a posteriori estimation," in *Position Location and Navigation Symposium (PUNS 'PO). pp. 464469,* March, 1990.

[11] E. J. Krakiwsky, "Tracking the worldwide development of IVHS navigation systems," *GPS World.* pp. 40–47, October, 1993.

[12] H. B. Lee, "A novel procedure for assessing the accuracy of hyperbolic multilateration systems," IEEE *Transactions on Aerospace and Electronic Systems*, vol. AES-11, no. 1, pp. 2–15, January, 1975.

[13] *A* Papoulis, *Probability, Random Variables, and Stochastic Processes*. McGraw–Hill, 1965.

[14] C. A Scott, "An overview of the communication options for broadcast DGPS corrections," *Australian Journal of Geodesy, Photogrametry, and Surveying*, vol. 58. pp. 69-84, June, 1993.

[15] *C.* A Scott, ***Improved** Positioning of Motor Vehicles Through Secondary Information Sources.* PhD thesis, University of Technology, Sydney, 1994. In preparation.

[16] C. A Scott and C. R. Drane, "Increased accuracy of motor vehicle position estimation by utilising map data, vehicle dynamics, and other information sources," in *Vehicle Navigation Information Systems Conference (VNIS '94)*, August, 1994.

[17] C. A Scott and C. R. Drane, "The chaotic streets of Sydney - the application of chaotic filtering to vehicle monitoring," in *The Third International Symposium on Signal Processing and its Applications (ISSPA 92)*, pp. 650–653, 1992.

[18] H W. Sorenson, ed., *Kalman Filtering: Theory and Application. IEEE* Press, 1985.

[19] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-20, *no.* 2, pp. 183-198, March, 1984.