Dynamic GPS Attitude Performance Using INS/GPS Reference

J. Chris McMillan
Defence Research Establishment Ottawa

D.A.G. Arden
Dale Arden Consulting

G. Lachapelle, G. Lu
University of Calgary

Abstract - A system for determining dynamic heading and attitude was installed on a 75 metre research vessel, the Endeavour, for performance evaluation. This system uses three or four independently operated, PC based, GPS cards (NovAtel GPSCard™ model 95lR) each with its own antenna. These are 10-channel, narrow-cot-relator-spacing receivers. The antennas were installed with about a 9 meter port-starboard separation, a 36 meter fore-aft separation and a 6 meter vertical separation. Seven days of data were collected at sea at a one Hz rate, and several hours of data were collected at 10 Hz. Data was also collected from two inertial navigators at 12.5 Hz, and from DGPS at one Hz. From these, two nominally independent (in attitude), optimally integrated INS/DGPS solutions were produced. Using these as reference, the accuracy performance of the GPS attitude determining system is evaluated, with error statistics presented and a linear stochastic error model derived. Of particular interest is the dynamic performance at higher rates (10 Hz) and longer baselines (36 meters),

INTRODUCTION

Sea trials were conducted in August 1992 and October 1993 by the Navigation Group of the Defence Research Establishment Ottawa. One objective was to evaluate GPS attitude measurement performance, for use on maritime platforms, to replace costly INSs (inertial navigation systems), or at least to complement them by allowing the use of lower cost INSs within an integrated system.

In comparison to an INS, a GPS attitude measuring receiver has many advantages, such as being relatively small, light and inexpensive, not requiring a long settling time and not being degraded by high latitude. However GPS also has disadvantages, such as it's susceptibility to loss of signal and much lower data rate.

For these and other reasons, GPS attitude is highly complementary to inertial, suggesting that their integration, even with a lower cost AHRS (attitude and heading reference system), may be highly advantageous, as discussed by McMillan and Ardcn [l]. The performance of such an integrated system will of course depend upon the individual INS and GPS error characteristics.

The purpose of this paper is tbercfore to describe and model the attitude errors of a GPS attitude determining system in the marine environment. This paper extends to longer baseline length and higher update rates, the results presented in reference [2] which describes the performance of two commercial receivers: the Ashtcch 3DF and the Trimble TANS Vector. The system under study here was developed at The University of Calgary and is based on independently operated, PC based, GPS cards (NovAtel GPSCard™ model 951R) each with its own antenna (model 501) and chokering groundplane. This system records the raw carrier phase measurements and post-processes them through the U. of Calgary MULTINAV™ software, as described in [3]. This software resolves the relative carrier ambiguities on the fly and estimates the attitude parameters independently at each point.

BACKGROUND

The determination of heading and attitude from GPS carrier phase measurements has been discussed extensively in the literature [e.g. 8,9,10]. Reference [4] analyzes in detail the performance of the 1992 3DF data and derives from it a stochastic error model for Kalman filter use.

An important factor to recall is that, all else being equal, a given differential phase measurement error $\delta\varphi$, produces an angular measurement error $\delta\theta$ which varies inversely with the antenna baseline length *d*, according to:

$$\delta\theta = \left(\frac{\lambda}{2\pi d \cos\theta}\right)\delta\varphi, \qquad (1)$$

Presented at ION GPS'94, Salt Lake City, September 21 - 23, 1994

the INS performance, this DGPS position data was integrated with each INS using Kalman filter software to produce estimates of the INS attitude errors (among other things), with covariance information to indicate the expected accuracy of these estimates. Since the errors in the attitude estimates from these two INS/DGPS solutions arc largely uncorrelated, their close agrcemcnt, as presented below, provides a high degree of confidence in heir accuracy.

The resolutions, data rates and expected accuracies of the relevant systems are listed in Table 2, where θ is heading, ϕ is pitch, ψ is roll and $\varepsilon \cong 10^{-16}$. The expected U. of C. GPS error shown here is based on the discussion above, extrapolating the 3DF and Vector data of Table 1 with the appropriate baseline lengths. This does not however include the GPS installation misalignment errors or data latency errors, both of which can be quite significant at this level of performance. The INS/GPS performance is discussed below.

Table 2. Expected System Performance

SENSOR	RESOLUTION (degrees)		DATA RATE	EXPECTED RMS ERROR (degrees)		
	θ	φ, ψ	(Hz)	θ	φ	ψ
GPS	ε	ε	1-10	≅0.02	≅0.02	go.05
INS	0.005	0.003	12.5	<0.07	co.03	<0.03
INS/DGPS	ε	ε	12.5	<0.01	<0.005	co.005

It should be mentioned that the INS data rate was intentionally limited due to data recording capacity (for the one week trial period) and not because of any INS limitation. The GPS attitude was only recorded at the high rate (10 Hz) for a few short periods (several hours in total), also because of data recording limitations, and because this was adequate for error modeling purposes.

REFERENCE SYSTEM ACCURACY

The expected INS/DGPS accuracy is based on a comparison of the attitude results from two "independent" INS/DGPS solutions, using INS 1 and INS2. These filtered solutions were generated using experimental software, developed at DREO for DIINS (the Dual Inertial Integrated Navigation System, de-scribed in reference [6]). Figure 3 illustrates the measured differences in the two DIINS heading estimates, (scaled by $1/\sqrt{2}$ to represent the errors of each individual solution), along with the DIINS prediction of it's own heading accuracy (± one sigma, or 68%). This prediction comes from the covariance of the Kalman filter's heading error state. Figures 4 and 5 show the same for pitch and roll.

These predicted and measured values are in good agreement. as seen from Table 3, where the measured standard deviations arc based on 646,000 data points (14 hours of data at 12.5 Hz). It should bc mentioned that the small misalignments between the two INSs have been left in Figures 3-5 so as not to hide the covariance.

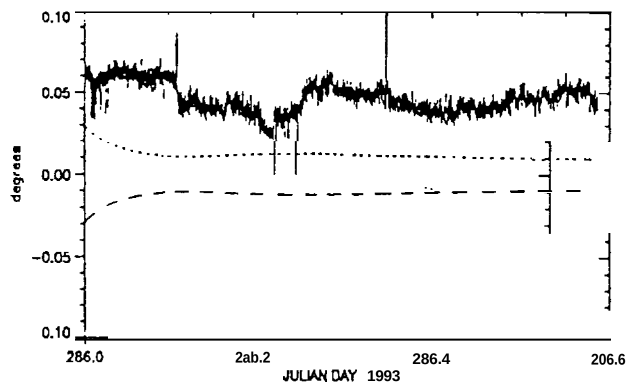


Figure 3. Reference Heading Difference & Covariance

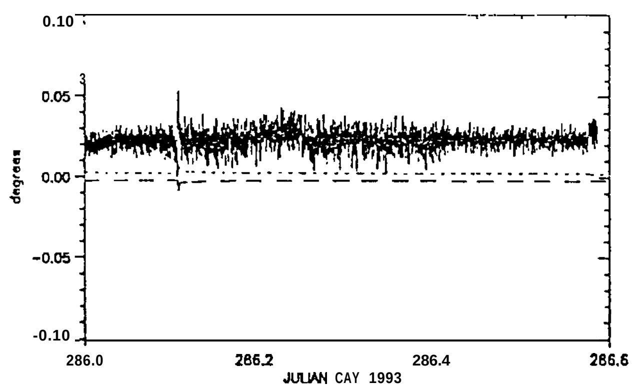


Figure 4. Reference Pitch Difference & Covariance

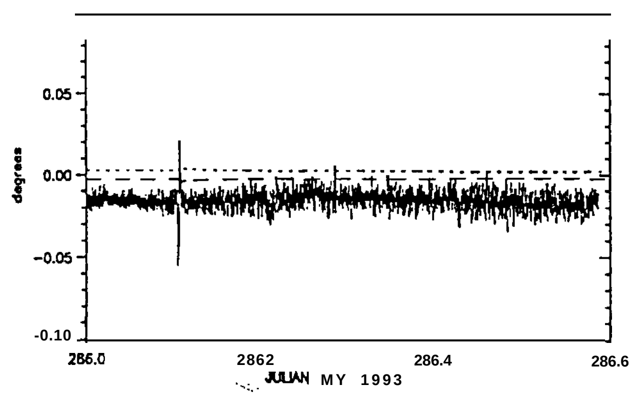


Figure 5. Reference Roll Difference & Covariance

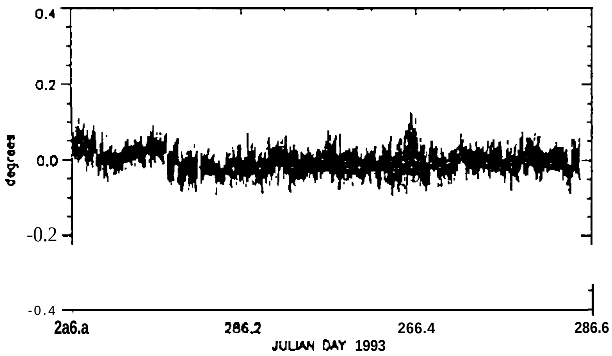


Figure 9. Discrete GPS Heading Error VS Time

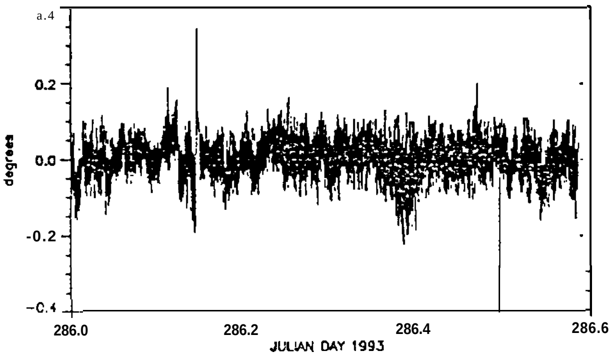


Figure 10. Discrete GPS Pitch Error VS Time

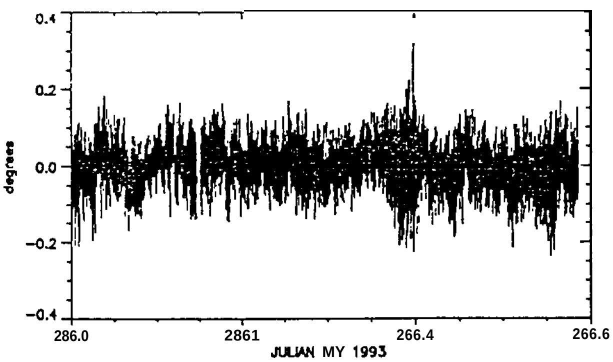


Figure 11. Discrete GPS Roll Error VS Time

Table 4. U. of Calgary Attitude Error at 1 Hz.

	STANDARD DEVIATION (degrees)	PERCENTILE (degrees)		
		68%	95%	99%
HEADING	0.026	0.026	**0.052**	0.07
PITCH	0.042	0.040	**0.084**	0.12
ROLL	0.049	0.048	**0.097**	0.13

These measured performance results arc based on 5 1,260 samples (over 14 hours) and show excellent consistency between the standard deviations, 68% and 95% values. These results arc also quite close to the predictions in Table 2, which were based on an extrapolation of the 3DF and Vector results of Table 1, taking into account the different baseline lengths. The fact that the longer baseline components (heading and pitch) are not quite as accurate as a linear extrapolation would predict, suggests that the differential phase measurement errors increase somewhat with baseline length (moslty duc to greater multipath decorrelation).

ERROR BEHAVIOUR AT 10 Hz

Since higher data rates are necessary for many applications, there is considerable interest in the performance of GPS attitude at 10 Hz. Figures 12-14 and Table 5 show the errors and their statistics for the 10 Hz altitude data, based on 50,326 samples over about 1.4 hours. Since this 10 Hz data had a significant amount of spurious data (about 1%), the standard deviation is also given wirh spurious data removed. Comparing this to the 1 Hz statistics in Table 4, it can be seen that the basic accuracy is retained at the higher rate.

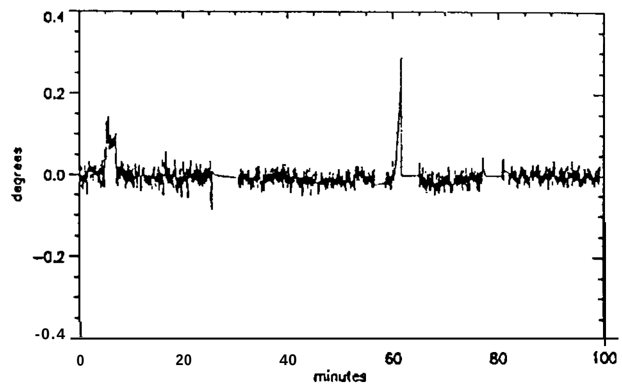


Figure 12. GPS Heading Error at 10 Hz

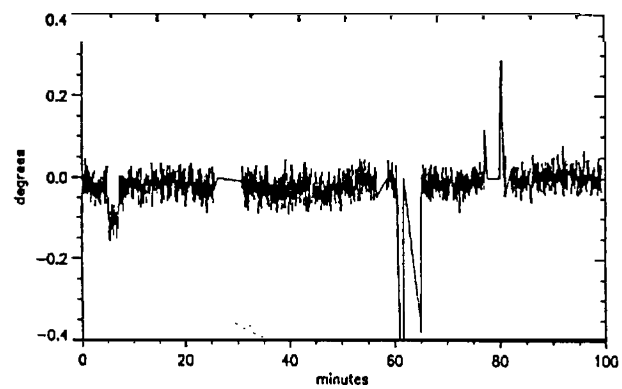


Figure 13. GPS Pitch Error at 10 Hz

Table 6. Vector/3DF Error Model Parameters

	NOISE	MARKOV	
	σ_1 (degrees)	σ_2 (degrees)	τ_2 (seconds)
HEADING	0.3/*d**	0.22/*d*	4,000
PITCH	0.4/*d*	0.35/*d*	***600***
ROLL	0.4/*d*	0.35/*d*	600

* ***d*** is the nominal **antenna** baseline length in meters

The complete sample autocorrelation functions of the discrete attitude errors of the 1 Hz U. of Calgary GPS data indicate that there is no significant autocorrelation for $n\Delta t$ > 8,000 seconds. Figures 15-17 show the first 8,000 seconds of these functions, from which it can be seen that there is a significant component of temporally correlated error. In fact, these sample autocorrelation function plots appear to be very closely matched to the correlation function of a first order Markov process (exponentially decaying as in equation (4)). Closer examination of the data however, reveals that there are strong peaks at At = 0, corresponding to an uncorrelated noise component.

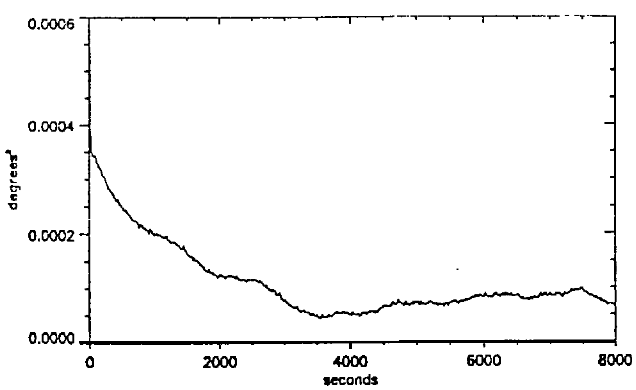


Figure 15. Heading Error Autocorrelation Function

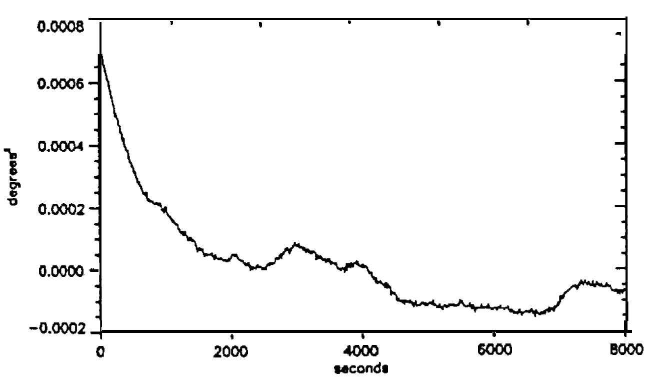


Figure 16. Pitch Error Autocorrelation Function

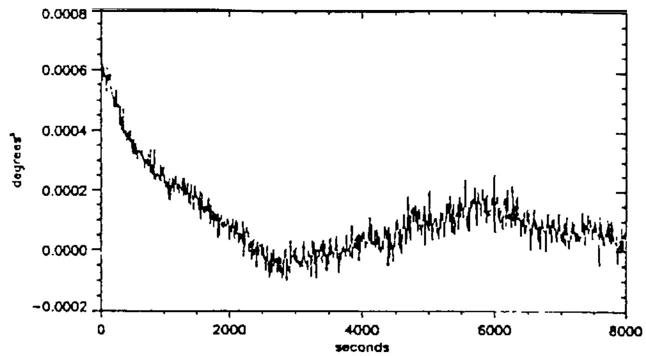


Figure 17. Roll Error Autocorrelation Function

If the error of each attitude component is assumed to be the sum of two independent stationary processes:

$$\delta\theta = \delta\theta_1 + \delta\theta_2 \qquad (9)$$

where $\delta\theta_1$ is white noise and $\delta\theta_2$ is a Markov process, then it can be shown that the autocorrelation function for $\delta\theta$ is simply the sum of the autocorrelation functions of $\delta\theta_1$ and $\delta\theta_2$. Since this generalizes to the sum of *n* independent processes, the autocorrelation function can be used to decompose the error into component parts.

Since the autocorrelation function of a white noise process is zero except at At = 0, the peaks at At = 0 are due to the white noise components and can be removed to find the model parameters for the Markov component. The magnitude of the peaks ***above the exponentially decaying portion,*** in Figures 15-17, are therefore the mean square of the white noise component. Although these peak values at At = 0 cannot be seen from these figures, they can be easily obtained from the data files. This error model parameter extraction process is discussed below for the heading error, and the results are summarized in Table 7 for all three attitude components.

Table 7. U. of Calgary Error Model Parameters

	NOISE	MARKOV	
	σ_1 (degrees)	σ_2 (degrees)	72 (seconds)
HEADING	0.018	0.019	2,000
PITCH	0.032	0.027	600
ROLL	0.042	0.025	1,000

From the sample autocorrelation data file, the total σ^2 (= $\sigma_1^2 + \sigma_2^2$) for the heading error is $\varphi_{\theta\theta}(0)$ = 0.00070 deg.^{2}. From Figure 1.5 the Markov σ_2^2 is about 0.00036 deg.^{2}. This puts the white noise σ_1^2 estimate at 0.0034

Dynamic GPS Attitude Performance Using INS/GPS Reference

J. Chris McMillan
Defence Research Establishment Ottawa

D.A.G. Arden
Dale Arden Consulting

G. Lachapelle, G. Lu
University of Calgary

Abstract - A system for determining dynamic heading and attitude was installed on a 75 metre research vessel, the Endeavour, for performance evaluation. This system uses three or four independently operated, PC based, GPS cards (NovAtel GPSCard™ model 951R) each with its own antenna. These are IO-channel, narrow-cot-relator-spacing receivers. The antennas were installed with about a 9 meter port-starboard separation, a 36 meter fore-aft separation and a 6 meter vertical separation. Seven days of data were collected at sea at a one Hz rate, and several hours of data were collected at 10 Hz. Data was also collected from two inertial navigators at 12.5 Hz, and from DGPS at one Hz. From the-se, two nominally independent (in attitude), optimally integrated INS/DGPS solutions were produced. Using these as reference, the accuracy performance of the GPS attitude determining system is evaluated, with error statistics presented and a linear stochastic error model derived. Of particular interest is the dynamic performance at higher rates (10 Hz) and longer baselines (36 meters).

INTRODUCTION

Sea trials were conducted in August 1992 and October 1993 by the Navigation Group of the Defence Research Establishment Ottawa. One objective was to evaluate GPS attitude measurement performance, for use on maritime platforms, to replace costly INSs (inertial navigation systems), or at least to complement them by allowing the use of lower cost INSs within an integrated system.

In comparison to an INS, a GPS attitude measuring receiver has many advantages, such as being relatively small, light and inexpensive, not requiring a long settling time and not being degraded by high latitude. However GPS also has disadvantages, such as it's susceptibility to loss of signal and much lower data rate.

For these and other reasons, GPS attitude is highly complementary to inertial, suggesting that their integration, even with a lower cost AHRS (attitude and heading reference system), may be highly advantageous, as discussed by McMillan and Arden [I]. The performance of such an integrated system will of course depend upon the individual INS and GPS error characteristics.

The purpose of this paper is therefore to describe and model the attitude errors of a GPS attitude determining system in the marine environment. This paper extends to longer baseline length and higher update rates, the results presented in reference [2] which describes the performance of hvo commercial receivers: the Ashtech 3DF and the Trimble TANS Vector. The system under study here was developed at The University of Calgary and is based on independently operated, PC based, GPS cards (NovAtel GPSCard™ mode1 951R) each with its own antenna (model 501) and chokering groundplane. This system records the raw carrier phase measurements and post-'processes them through the U. of Calgary MULTINAV™ software, as described in [3]. This software resolves the relative carrier ambiguities on the fly and estimates the attitude parameters independently at each point.

BACKGROUND

The determination of heading and attitude from GPS carrier phase measurements has been discussed extensively in the literature [e.g. 8,9,10]. Reference [4] analyzes in detail the performance of the 1992 3DF data and derives from it a stochastic error model for Kalman filter use.

An important factor to recall is that, all else being equal, a given differential phase measurement error $\delta\varphi$, produces an angular measurement error $\delta\theta$ which varies inversely with the antenna baseline length d, according to:

$$\delta\theta = \left(\frac{\lambda}{2\pi d \cos\theta}\right)\delta\varphi, \tag{1}$$

where λ is the wavelength (so that $\lambda\delta\varphi/2\pi$ is the differential range error). Thus performance can generally be improved by reducing the measurement error $\delta\varphi$, or extending the baseline length ***d***. Unfortunately there are often platform size limitations, not to mention mechanical stability limitations which preclude the use of extremely long baselines.

Another method of improving accuracy is to extend the sample period, using some averaging or filtering technique to reduce the phase measurement error. This is only possible when the attitude can be assumed constant, as in the static situation, or if inertial-type aiding is available.

Reference [2] describes the two commercial systems used in the 1992 and 1993 trials (an Ashtech 3DF and a Trimble TANS Vector respectively) and presents some performance test results. Both systems used 4-antenna arrays, with installation geometries as illustrated in Figure 1. The most important factor is the baseline lengths, which were about 10 metres in the case of the 3DF and 2 metres for the Vector.

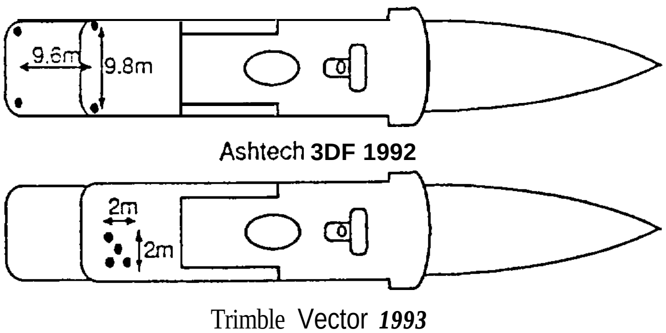


Figure 1. GPS Antenna Arrays (View From Above)

The basic dynamic error statistics from these two systems, as measured on the Endeavour, are summarized in Table 1, where data latency errors are ignored and the constant misalignment errors have been removed.

Table 1. GPS Attitude Errors (bias removed)

		STANDARD **DEVIATION** (degrees)	PERCENTILE (degrees) 68%	95%
3DF:	HEADING	0.47	0.04	0.08
	PITCH	0.27	0.05	0.11
	ROLL	0.26	0.05	0.11
Vector:	HEADING	0.36	0.15	0.39
	PITCH	0.36	0.16	0.42
	ROLL	0.30	0.15	0.31

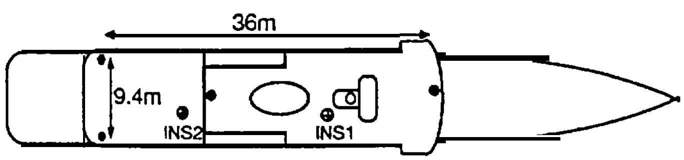
The standard deviation numbers shown in Table 1 are based on about 450,000 data points, taken at a one Hz rate over about one week, while the percentiles are based on samples taken every 20 seconds Over the same period. The relatively large standard deviations, as compared to the 68 percentiles (especially for the 3DF), are due to a fairly small amount (about 1%) of poor data.

Equation 1 above predicts that a 1 cm. differential range error would produce attitude errors on the order of about 0.05 degrees with a 10 metre baseline and 0.25 degrees with a 2 metre baseline (assuming ideal geometry, with $\cos\theta \cong 1$). This is generally consistent with the 68% results of Table 1, however it can be seen that the longer baseline did not quite produce the factor of 5 improvement in accuracy predicted by equation 1. This therefore suggest that the heading accuracy with a 36 metre baseline should be somewhat worse than 0.015 degrees (68%).

SEA TRIAL METHODOLOGY

These trials were conducted off the west coast of Canada on a 75 meter, 1600 ton research vessel, from August 18 to 24, 1992 and from Oct. 12 to 19, 1993. There were no indications of any unusual conditions of the GPS space or control segments during these trials.

The system of primary interest here, described in more detail in references [3] and [5], is a GPS based attitude measuring system developed at The University of Calgary using 3 or 4 independently operating PC-based receivers, each with a NovAtel GPSCard™ sensor, Model 951R, which is a lo-channel narrow-correlator spacing C/A code GPS receiver. Each unit also had a Model 501 antenna with chokering groundplane to reduce multipath. This U. of Calgary system was installed with a 36 metre heading/pitch baseline, as shown in Figure 2.



U. of Calgary 1993

Figure 2. GPS Antenna Array (View From Above)

The navigation sensor complement also included two marine inertial navigators, referred to here as INS1 and INS2. They provided heading, pitch and roll with sufficient accuracy to observe and quantify the GPS attitude errors of the short baseline GPS receivers described above, as discussed in [2]. For the U. of Calgary system however, this raw INS data is not quite adequate.

There was also a DGPS receiver providing position reference accurate to about 1 metres (lo) [11]. To verify the INS performance, this DGPS position data was integrated with each INS using Kalman filter software to produce estimates of the INS attitude errors (among other things), with covariance information to indicate the expected accuracy of these estimates. Since the errors in the attitude estimates from these two INS/DGPS solutions are largely uncorrelated, their close agreement, as presented below, provides a high degree of confidence in their accuracy.

The resolutions, data rates and expected accuracies of the relevant systems are listed in Table 2, where θ is heading, ϕ is pitch, ψ is roll and $\varepsilon \cong 10^{-16}$. The expected U. of C. GPS error shown here is based on the discussion above, extrapolating the 3DF and Vector data of Table 1 with the appropriate baseline lengths. This does not however include the GPS installation misalignment errors or data latency errors, both of which can be quite significant at this level of performance. The INS/GPS performance is discussed below.

Table 2. Expected System Performance

SENSOR	RESOLUTION (degrees)		DATA RATE	EXPECTED RMS ERROR (degrees)		
	θ	ϕ,	ψ (Hz)	θ	ϕ	ψ
GPS	ε	ε	1-10	≅0.02	≅0.02	≅ 0.05
INS	0.005	0.003	12.5	co.07	co.03	co.03
INS/DGPS	ε	ε	12.5	<0.01	<0.005	co.005

It should be mentioned that the INS data rate was intentionally limited due to data recording capacity (for the one week trial period) and not because of any INS limitation. The GPS attitude was only recorded at the high rate (10 Hz) for a few short periods (several hours in total), also because of data recording limitations, and because this was adequate for error modeling purposes.

REFERENCE SYSTEM ACCURACY

The expected INS/DGPS accuracy is based on a comparison of the attitude results from two "independent" INS/DGPS solutions, using INS 1 and INS2. These filtered solutions were generated using experimental software, developed at DREO for DIINS (the Dual Inertial Integrated Navigation System, described in reference [6]). Figure 3 illustrates' the measured differences in the two DIINS heading estimates, (scaled by $1/\sqrt{2}$ to represent the errors of each individual solution), along with the DIINS prediction of it's own heading accuracy (± one sigma, or 68%). This prediction comes from the covariance of the Kalman filter's heading error state. Figures 4 and 5 show the same for pitch and roll.

These predicted and measured values are in good agreement. as seen from Table 3, where the measured standard deviations are based on 646,000 data points (14 hours of data at 12.5 Hz). It should be mentioned that the small misalignments between the two INSs have been left in Figures 3-5 so as not to hide the covariance.

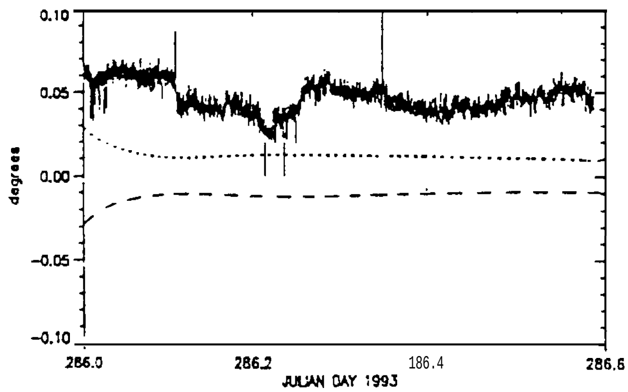


Figure 3. Reference Heading Difference & Covariance

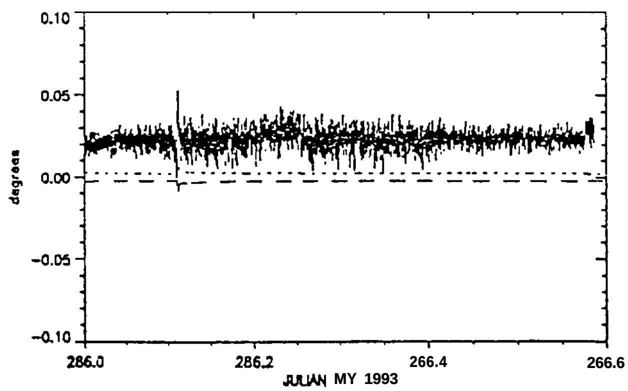


Figure 4. Reference Pitch Difference & Covariance

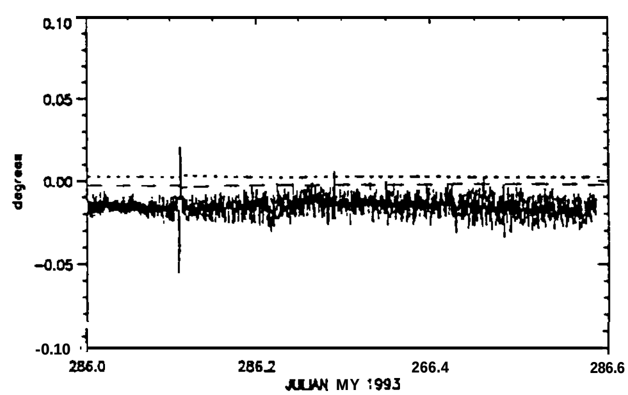


Figure 5. Reference Roll Difference & Covariance

Table 3. Reference System Performance

	Predicted (Covariance) (degrees)	Measured (Standard Deviation) (degrees)
Heading	0.011	<0.010
Pitch	0.003	<0.005
Roll	0.003	co.005

Table 3 therefore verifies that the INS/DGPS attitude data was sufficiently accurate to measure the expected level of GPS errors, as indicated in Table 2, although in the case of heading it is just barely sufficient

RAW DATA

The raw data (consisting of time-tagged, relative antenna position measurements) was post processed at The University of Calgary, using the MULTINAV^{TM} attitude software. This produced the attitude of the GPS antenna frame with respect to WGS84 frame. The constant, large angle, 3-dimensional rotation needed to transform this to provide attitude of the ships body frame was then computed at DREO, using the DIINS reference data. The U. of C. attitude data was then rotated appropriately. This procedure coincidentally removes any constant misalignment error between antenna frame and platform frame, in a way which relies on the ability of an inertial system to be precisely aligned (something that is very difficult to do with GPS antennas).

Figures 6, 7 and 8 show the U. of C. heading, pitch and roll (at 10 Hz) in comparison to the reference data (at '12.5 Hz), for a brief interval These illustrate the short term dynamics, due largely to wave motion, and at the same time give a first impression of the quality of the GPS data.

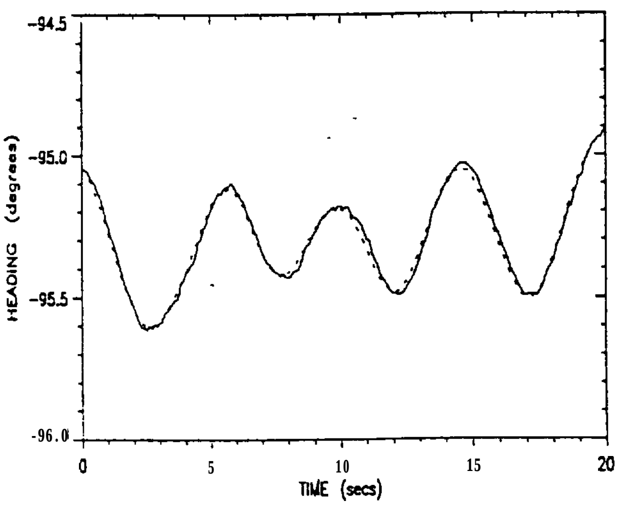


Figure 6. GPS (solid) & Reference (dashed) Heading

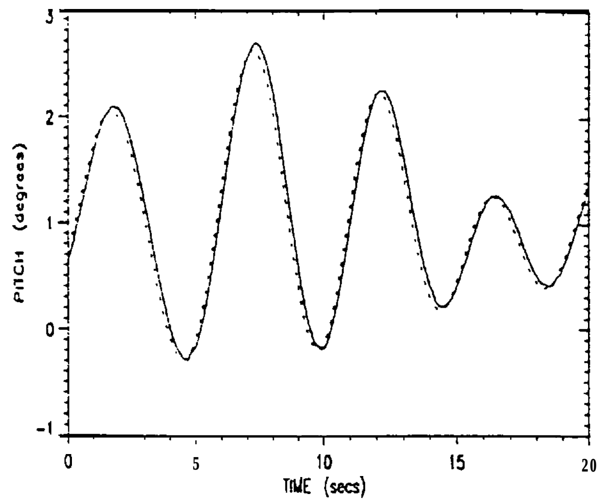


Figure 7. GPS (solid) & Reference (dashed) Pitch

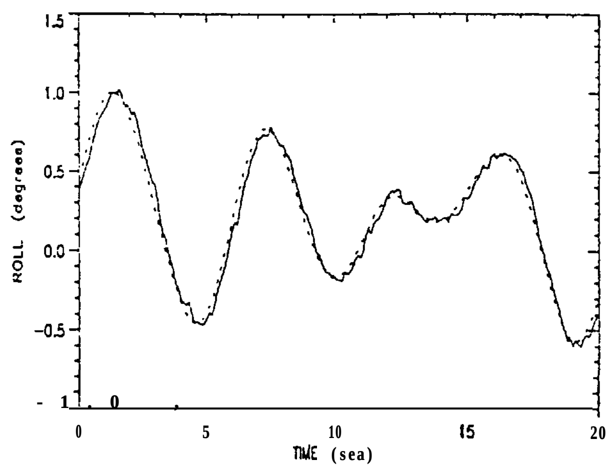


Figure 8. GPS (solid) & Reference (dashed) Roll

MEASURED GPS ERROR BEHAVIOUR

When dealing with the ***real-time*** dynamic performance of a system which only provides discrete output. care must be taken to define what errors are being examined. Since data interpolation is not possible in real time, there are additional errors to consider. However in this paper (as in most others) the errors due to changes in the attitude between measurements will be ignored. We therefore examine only the measurement errors at the GPS data record times. These will be referred to as ***discrete dynamic errors.*** They can be interpreted as the errors seen by a user such as an integrated system with an inertial component, which can provide the necessary real time extrapolation.

The discrete dynamic GPS attitude errors throughout the trial are shown in Figures 9-11. The corresponding statistics are presented in Table 4.

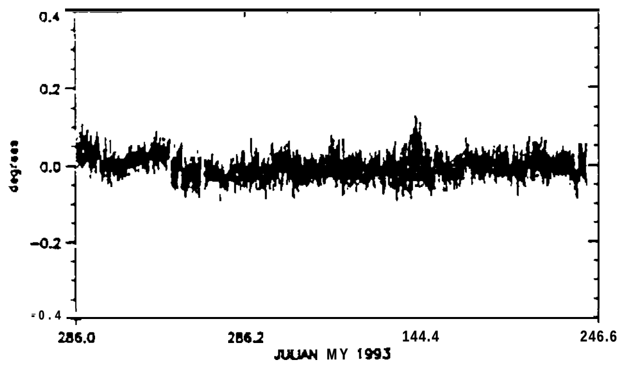


Figure 9. Discrete GPS Heading Error VS Time

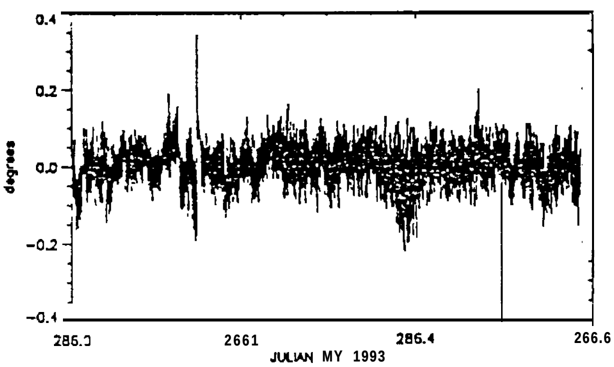


Figure 10. Discrete GPS Pitch Error VS Time

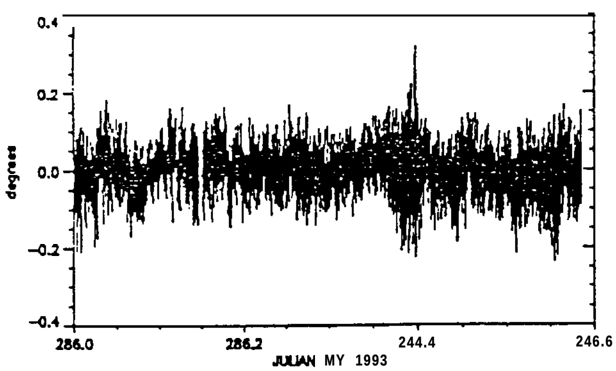


Figure 11. Discrete GPS Roll Error VS Time

Table 4. U. of Calgary Attitude Error at 1 Hz.

	STANDARD DEVIATION (degrees)	PERCENTILE (degrees) 68%	95%	99%
HEADNG	0.026	0.026	0.052	0.07
PITCH	0.042	0.040	0.084	0.12
ROLL	0.049	0.048	0.097	0.13

These measured performance results are based on 51,260 samples (over 14 hours) and show excellent consistency between the standard deviations, 68% and 95% values. These results are also quite close to the predictions in Table 2, which were based on an extrapolation of the 3DF and Vector results of Table 1, taking into account the different baseline lengths. The fact that the longer baseline components (heading and pitch) are not quite as accurate as a linear extrapolation would predict, suggests that the differential phase measurement errors increase somewhat with baseline length (moslty due to grater multipath decorrelation).

ERROR BEHAVIOUR AT 10 Hz

Since higher data rates are necessary for many applications, there is considerable interest in the performance of GPS attitude at 10 Hz. Figures 12-14 and Table 5 show the errors and their statistics for the 10 Hz attitude data, based on 50,326 samples over about 1.4 hours. Since this 10 Hz data had a significant amount of spurious data (about 1%), the standard deviation is also given with spurious data rcmovcd. Comparing this to the 1 Hz statistics in Table 3, it can be seen that the basic accuncy is retained at the higher rate.

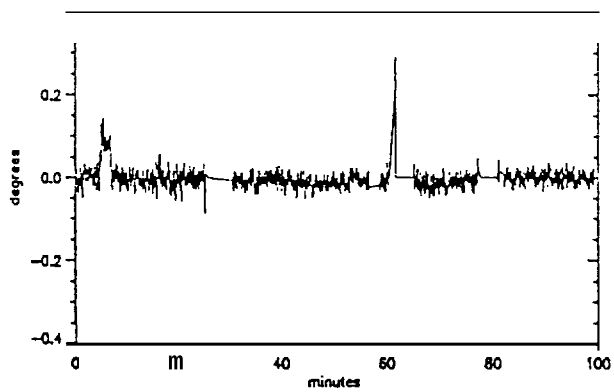


Figure 12. GPS Heading Error at 10 Hz

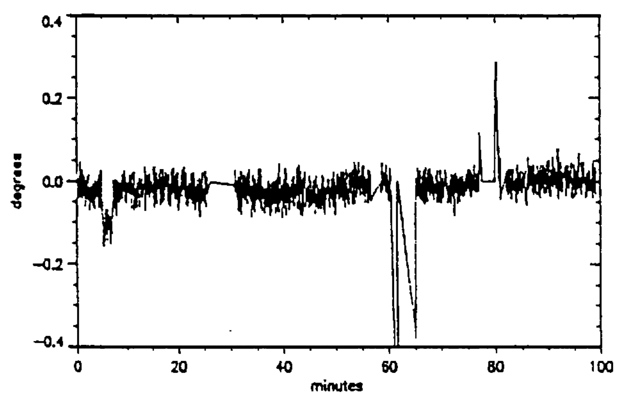


Figure 13. GPS Pitch Error at 10 Hz

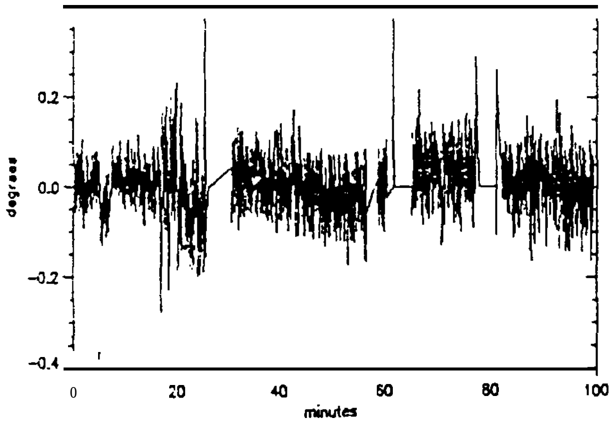


Figure 14. GPS Roll Error at 10 Hz

Table 5. U. of Calgary Attitude Error at 10 Hz.

	STANDARD DEVIATION (degrees)		PERCENTILE (degrees)		
	raw	edited	68%	95%	99%
HEADING	0.30	0.021	0.015	0.037	0.23
PITCH	0.42	0.037	0.027	0.063	0.21
ROLL	0.92	0.057	0.053	0.117	1.10

STOCHASTIC ERROR MODEL

Stochastic error models will now be developed for each attitude error component. Besides offering insight into the error behaviour of these measurements, the models will be in a form suitable for use by a Kalman filter [1]. This model will be developed using the same technique used in reference [2], where such an error model was developed for the 3DF and Vector attitude errors. The method used was simply to match the measured "sample autocorrelation function" of each attitude error component, to the parametric form of a standard autocorrelation function, thereby extracting the necessary parameter values.

The stochastic models normally used by Kalman filter designers to describe random variables are simple linear models such as a random bias, white noise, Markov process, random walk or periodic process. These are adequately described in reference [7] and are usually distinguished by their distinctive autocorrelation functions $\varphi(\Delta t)$. The autocorrelation function $\varphi_{xx}(\Delta t)$, of a random variable $x(t)$, is defined to be the expected correlation between values of $x(t)$ separated in time by At seconds:

$$\varphi_{xx}(\Delta t) \equiv E\{x(t)x(t+\Delta t)\} \tag{3}$$

Perhaps the most useful stochastic model is the first order Markov process, since the random bias and white noise are both, in some sense, a special case of this Markov process. The Markov process x(t) is described by two parameters: its rms value σ and its autocorrelation time τ. It has the following autocorrelation function:

$$\varphi_{xx}(\Delta t) = \sigma^2 e^{-\Delta t/\tau} \tag{4}$$

If τ is very large, this Markov process will essentially behave as a random bias, and if τ is very small it will behave as white noise. Equation (4) therefore provides an ideal "template" to use in extracting the error model parameters (a and τ) from an autocorrelation function.

Now the definition given by equation (3) can be used to obtain a "sample autocorrclation function" from the measured $x(t)$ (the GPS attitude errors in this case). Then by matching the plot of this autocorrelation function φ_{xx} to the template of equation (4), the model parameters σ and τ can be easily extracted as follows. The initial value is σ:

$$\sigma^2 = \varphi_{xx}(0) \tag{5}$$

and the point where φ_{xx} drops to σ^2/e is τ:

$$\varphi_{xx}(\tau) = \sigma^2 e^{-1} \tag{6}$$

Autocorrelation Functions

As described in reference [7], the sample autocorrelation function φ_{xx} of the variable x_n, n=1...N (discrete samples of a stochastic process X(I) at times $n\Delta t$ where $x(t)$ may be a vector) is defined to be:

$$\varphi_{xx}(n\Delta t) \equiv \frac{1}{N-n-1}\sum_{i=1}^{N-n}(x_i - m)(x_{i+n} - m)^T$$
$$n=0, 1, \ldots, N-2 \tag{7}$$

where m is the sample mean:

$$m = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{8}$$

In reference [2] it was shown that the 3DF and Vector attitude errors both closely matched the model given in Table 6. It was also shown that, given the large sample size ($\cong$ 500,000 data points), these parameter values should be accurate to about 5% (assuming the model template was valid).

Table 6. Vector/3DF Error Model Parameters

	NOISE	MARKOV	
	σ_1 (degrees)	σ_2 (degrees)	τ_2 (seconds)
HEADING	$0.3/d$*	$0.22/d$	4,000
PITCH	$0.4/d$	$0.35/d$	*600*
ROLL	$0.4/d$	$0.35/d$	600

* *d is the* nominal antenna baseline length in meters

The complete sample autocorrelation functions of the discrete attitude errors of the 1 Hz U. of Calgary GPS data indicate that there is no significant autocorrelation for $n\Delta t$ > 8,000 seconds. Figures 15-17 show the first 8,000 seconds of these functions, from which it can be seen that there is a significant component of temporally correlated error. In fact, these sample autocorrelation function plots appear to be very closely matched to the correlation function of a first order Markov process (exponentially decaying as in equation (4)). Closer examination of the data however, reveals that there are strong peaks at At = 0, corresponding to an uncorrelated noise component.

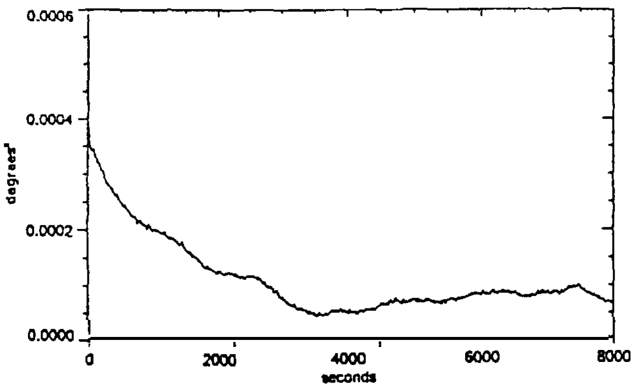


Figure 15. Heading Error Autocorrelation Function

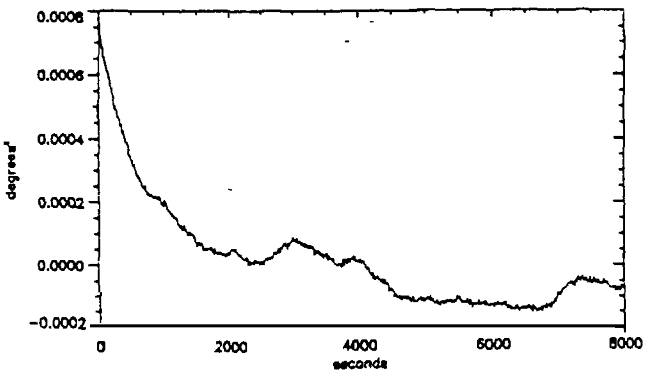


Figure 16. Pitch Error Autooorrelation Function

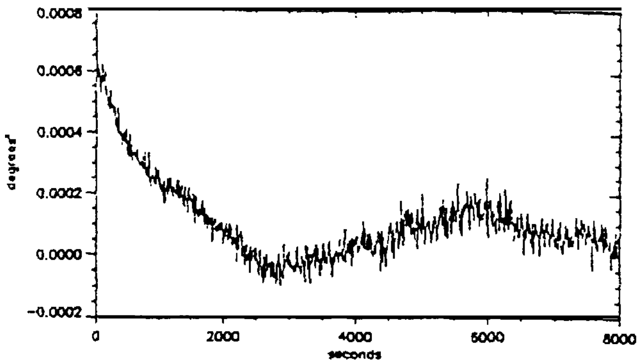


Figure 17. Roll Error Autocorrelation Function

If the error of each attitude component is assumed to be the sum of two independent stationary processes:

$$\delta\theta = \delta\theta_1 + \delta\theta_2 \qquad (9)$$

where $\delta\theta_1$ is white noise and $\delta\theta_2$ is a Markov process, then it can be shown that the autocorrelation function for $\delta\theta$ is simply the sum of the autocorrelation functions of $\delta\theta_1$ and $\delta\theta_2$. Since this generalizes to the sum of n independent processes, the autocorrelation function can be used to decompose the error into component parts.

Since the autocorrelation function of a white noise process is zero except at At = 0, the peaks at At = 0 are due to the white noise components and can be removed to find the model parameters for the Markov component. The magnitude of the peaks above *the exponentially decaying portion,* in Figures 15-17, are therefore the mean square of the white noise component. Although these peak values at At = 0 cannot be seen from these figures, they can be easily obtained from the data files. This error model parameter extraction process is discussed below for the heading error, and the results are summarized in Table 7 for all three attitude components.

Table 7. U. of Calgary Error Model Parameters

	NOISE	MARKOV	
	σ_1 (degrees)	σ_2 (degrees)	τ_2 (seconds)
HEADING	0.018	0.019	2,000
PITCH	0.032	0.027	600
ROLL	0.042	0.025	1,000

From the sample autocorrelation data file, the total σ^2 (= $\sigma_1^2 + \sigma_2^2$) for the heading error is $\varphi_{\theta\theta}(0)$ = 0.00070 deg.^{2}. From Figure 15 the Markov σ_2^2 is about 0.00036 deg.^{2}. This puts the white noise σ_1^2 estimate at 0.0034 deg.^{2}. The Markov correlation time is the point where the Markov φ drops below $\sigma_2^2/e = 0.00036/2.718 = 0.00013$, which is at about $\tau_2 = 2{,}000$ seconds. The error model parameters for pitch and roll are found in the same way. and the results arc as shown in Table 7.

Comparing this error model to that for the 3DF and Vector data sets, as shown in Table 6, several observations can be made. For each attitude component of each system, the uncorrelated noise and the correlated error are both of about the same magnitude ($\sigma_1 \equiv \sigma_2$). The correlation times for the different systems are in general agreement, with the heading error having longer correlation time. In fact the model of Table 6, with it's functional dependence on antenna baseline length, fits this U. of C. data quite well if allowance is made for a slight increase in differential phase measurement error (due to increased differential multipath) when very long antenna baselines are used (in this case the heading/pitch baseline).

It should also be kept in mind that these *discrete* errors do not include the latency or interpolation errors that would be present (and very significant) in the continuous dynamic situation with a stand alone GPS system.

A simple interpretation of this mode! would be that the Markov components were due to differential multipath errors, the white noise components due to receiver carrier phase measurement noise and the bias due to installation alignment (calibration) error. (Differential antenna phase center migration may also contribute to the Markov error.)

CONCLUSIONS

It has been demonstrated here, by direct measurement, using an accurate and independent reference system, that the GPS heading measurements can be made in a dynamic mode, at 10 Hz, with an accuracy of better than 0.05 degrees 95% (less than one mil).

It is observed that the attitude accuracy does not quite increase linearly with baseline length. This is perhaps due to multipath errors canceling (when the differential phase measurements are taken) to a greater extent with short baselines.

Another important observation is that the higher data rate (10 Hz) does not seem to effect the accuracy.

This data clearly demonstrates the potentia! of this type of GPS receiver to provide very accurate dynamic attitude information. This is especially true in the static case if multipath can be avoided and in the dynamic case if inertia! aiding is available.

The statistics for the discretedynamic attitude errors (at the GPS data points), with bias removed, are given in Tables 4 and 5, and the dominant stochastic error model parameters, for the white noise and Markov components, are shown in Table 7. Comparing this U. of Calgary data to the 3DF and Vector data (Tables 1 and 6) confirms that the error characteristics are not strongly receiver dependent.

These observations can be given with confidence, since the errors of the reference system (optimally integrated INS/DGPS) were shown to be sufficiently small.

References

[1] J.C. McMillan, D.A.G. Arden, "Sensor Integration Options for Low Cost Position & Attitude Determination," Proceedings of the Position Location and Navigation Symposium, Las Vegas, April 11-15, 1994.

[2] J.C. McMillan, "A GPS Attitude Error Mode! for Kalman Filtering," Proceedings of the Position Location and Navigation Symposium, Las Vegas, April 1 I-15, 1994.

[3] G. Lu, G. Lachapelle, M.E. Cannon, P. Kielland, "Attitude Determination in a Survey Launch Using Multi-Antenna GPS Technology," Proceedings of the ION 1993 National Technical Mccting, San Francisco, January 29-22, 1993.

[4] J.C. McMillan, "Intcrferometric GPS Attitude: A Stochastic Error Mode!," Report No. 1168, Dcfencc Research Establishment Ottawa, February 1993.

[5] G. Lachapelle, G. Lu, B. Loncarevic, "Precise Shipbome Attitude Determination Using Wide Antenna Spacing," Proceedings of the International Symposium on Kinematic Systems in Geodesy, Geomatics and Navigation, Banff, Alberta, Aug. 30-Sept. 2, 1994.

[6] J.C. McMillan, J.S. Bird, D.A.G. Arden, "Techniques for Soft-Failure Detection in a Multisensor Integrated System", NAVIGATION, Journal Of The Institute Of Navigation, Vol. 40, No. 3, Fall 93

[7] *Applied Optimal Estimation*, edited by A. Gelb, M.I.T. Press, Massachusetts Institute of Technology, 1974.

[8] J. Kilvington, "A Proposal for a Navstar Interferometric Attitude Reference System," Royal Aircraft Estab lishment Tech Memo: Rad-Nav 5 1, April 1977.

[9] L.R. Kruczynski, P.C. Li, A.G. Evans, B.R. Herman, "Using GPS to Determine Vehicle Attitude: U.S.S. Yorktown Test Results," Proceedings of ION-GPS 89, pp. 163-171.

[10] F. Van Graas, M. Braasch, "GPS Interferometric Attitude and Heading Determination: Initial Flight Test Results." NAVIGATION, Journal of the Institute of Navigation, Vol. 38, No. 3, Winter 91-92.

[11] G. Lachapelle, R. Klukas, D. Roberts, W. Qui, J.C. McMillan, "One Meter Level Kinematic Point Positioning Using Precise Orbit and Timing Information," Proceedings of GPS-94, Salt Lake City, Sept 21-23, 1994.